*J.M.S* Engg. Mechanics

UNIT - 1
Basics of Mechanics and force system.

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Basics of Mechanics and force system.

chapter 1 (fundamental)
chapter 2 ( forces)
chapter 3. (Resolution of force)
chapter 4 (Moment of a force)
chapter 5 (composition of forces)

UNIT-2
Equilibrium

UNIT-3 Friction

UNIT-4 COG, CENTROID

UNIT-5 SIMPLE LIFTING MACHINE.

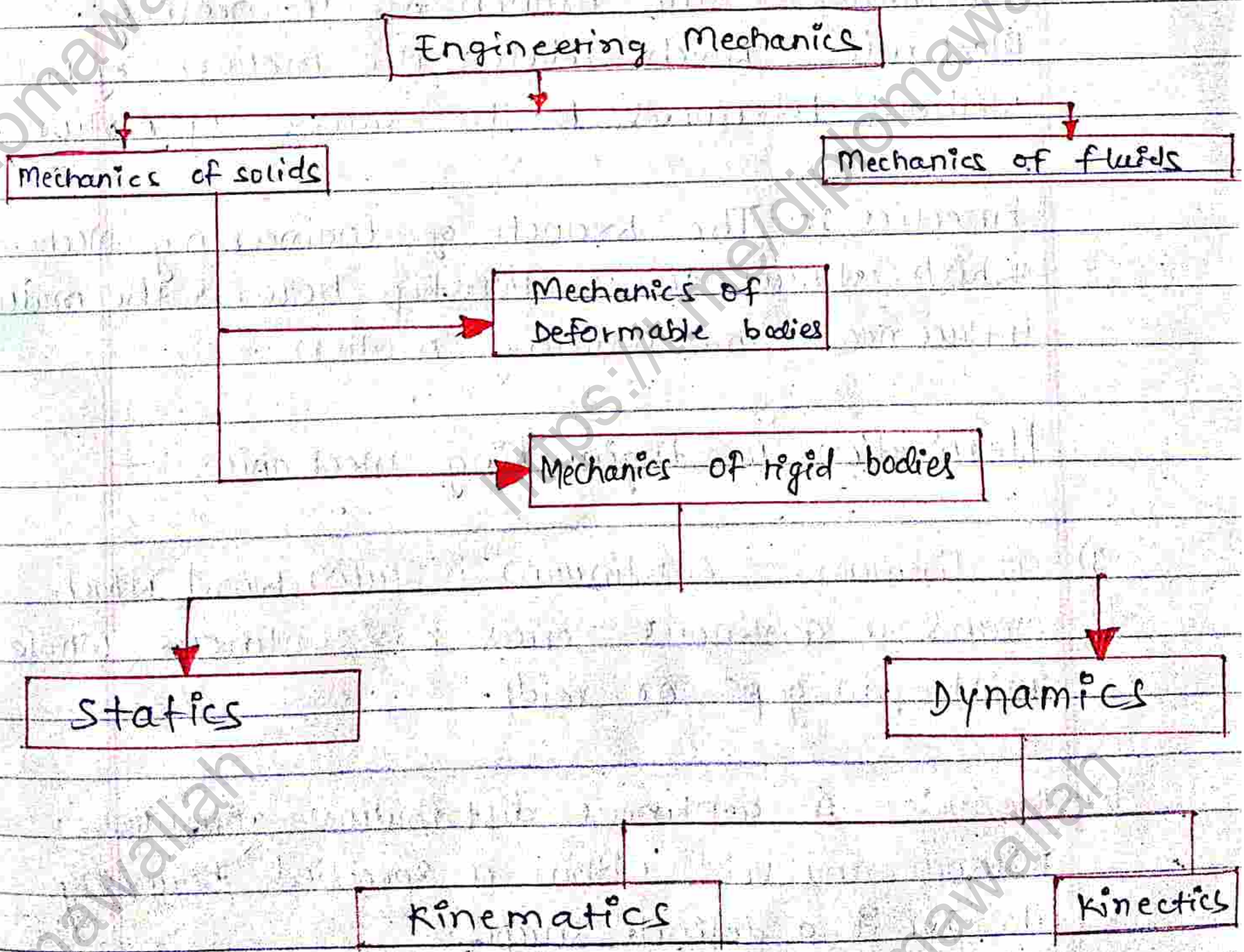
*J·M·S*

UNIT - 1

chapter - 1 **Fundamentals**

Engineering Mechanics — The branch of physical science which study of different laws and principles of mechanics applied to the engineering problem.

Mechanics - A branch of science which deals with the study of forces and their effects on bodies in motion or at rest.



Statics :- The branch of Engineering Mechanics which is concered with the conditions Under which bodies remain at rest relative to their surroundings."

Dynamics :- The branch of Engineering Mechanics which deals with the analysis of moving bodies.

Kinematics :- The branch of Engineering Mechanics which relates the motion of bodies without references to the cause of motion.

Kinecties :- The branch of Engineering Mechanics which studies the relationship between the mutual influcences and resulting motion.

Idealisation in Engineeering mechanics :-

a) Continuum :- continuum is latin word which means a continous series or continous whole with no gaps or voids.

b) Body :- A continous distribution of matter without any void within a specified boundary to give it a definite shape.

i) Rigid body :- A continuum which is not deformed under the effect of load i.e any two arbitrary points with in the body are undisturbed.


Basic Concepts :-

(a) Space :- Space is recognised as extending in every direction.
It is geometric extending or region which bodies occupy.

(b) Mass :- Mass is the characteristic of a body which represents the quantity of matter in a body and is a measure of the Inertia of a body.

(c) Weight :- The force with which the body gets attracted towards the centre of the earth by the action of gravitational pull.

$W = mg$. Where as,
W — weight
m — mass
g — gravitational acceleration
$9.8\ m s^{-2}$.

| Mass | Weight |
|---|---|
| It is quantity of matter contained in a body. | It is the force with which the body is attracted towards the centre of the earth. |
| It is constant for a body at all places. | It changes as place or orientation of a body changes. |
| It resist motion in the body | It supports motion in the body. |
| It is a scalar quantity | It is a vector quantity. |
| It is never zero. | It is zero at the centre of the earth. |
| It is measured in (kg) | It is measured in (N) |

(d) length :- length is a concept for describing size quantitatively.
The distance of seperation between any two points at the extreme ends of the object.

(e) Time :- A basic quantity in dynamics and it is not directly involved with the statics related problems.

(f) Scalars :- The quantity which have only magnitude :
Eg:- mass, volume, time, energy ... etc

(g) vectors :- The quantity which have magnitude as well as direction.
Eg :- displacement, velocity, acceleration ....etc

Unit of measurement -

(a) Unit - The physical quantity to express its measure a standard of that physical quantity.

Measure of physical quantity = n u.

Where as;
n - numerical value.
u - Unit of the quantity.

length of string = 8 m

$$\underset{n}{\uparrow}\ \ \underset{u}{\uparrow}$$

common system of units -

(i) f.p.s system (foot, pound, second)
(ii) c.g.s system (centimeter, gram, second)
(iii) m.k.s system (metre, kilogram, second)
(iv) S.I system.

Fundamental Unit :- The physical quantity which have independent unit as like mass, length, time, etc.

Eg :- Mass, length, time.

Derived Unit :- The units of measurement of all other physical quantities, which can be obtained from fundamental units are called derived unit.

Eg :- Force, work, power, Energy ....

$$\text{Unit of speed} = \frac{\text{Unit of distance (length)}}{\text{Unit of time (second)}}$$

$$\text{Speed} = ms^{-1}$$

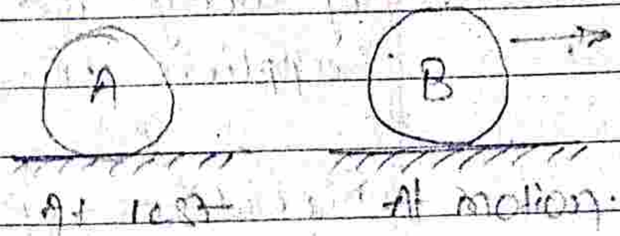
| Basic physical quantity | Fundamental unit | Symbol |
|---|---|---|
| Mass | Kilogram | Kg |
| length | metre | m |
| Time | Second | s |
| Temperature | Kelvin | K |
| Electric Current | Ampere | A |
| luminous intensity | candela | cd |
| Quantity of matter | mole | mol. |

| Supplementary physical Quantity | Supplementary Unit | Symbol |
|---|---|---|
| plane Angle | Radian | rad. |
| Solid Angle | Steradian | Sr. |

chapter-2 **Force**

• Force – It is the external agency that change the state of body.

• Rest to motion.
• motion to Rest.
• motion to retard.

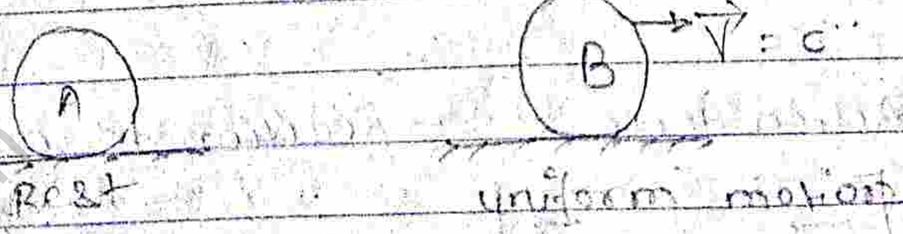



• Needed parameters to define force completely.
* Magnitude
* Direction
* point of application

• force is vector quantity.
• S.I unit of force is Newton (N).

## Newton's law

1• Newton's first law of motion → (NFL)
if the body is at rest / uniform motion then, state will remain same will or unless external force applied it.

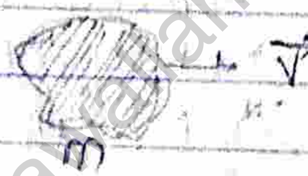



• This law is also called law of Inertia.
• Inertia is a physical power of any body.

2 * Newton second law of motion → (NSL) :-
The Rate of change of linear momentum of body is always equal to force applied on body.

- Direction of change in linear momentum will be same in the direction of force.

linear momentum, $\vec{p} = m\vec{v}$




$$\frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$$

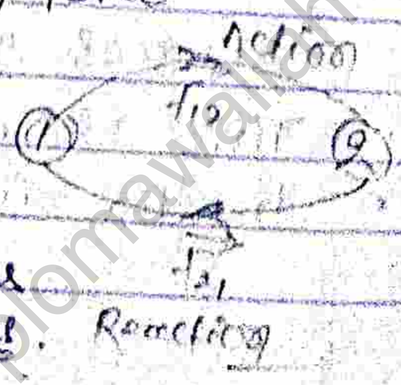
$$\vec{F} = m\vec{a} + 0$$

$$\boxed{\vec{F} = m\vec{a}}$$

3. Newton's third law of motion - (NTL)
To Every action force is equal and opposite reaction force.

Action force = − Reaction force

$$\boxed{\vec{F}_{12} = -\vec{F}_{21}}$$

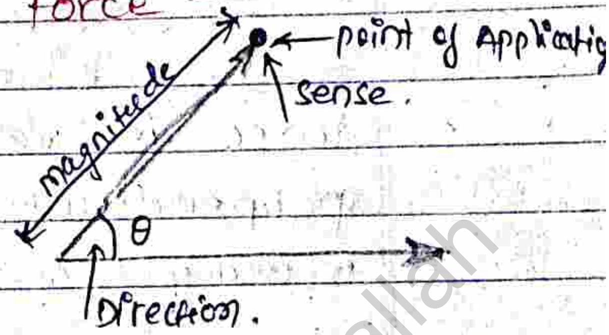



- Both action & Reaction forces are acting on different bodies.

* Newton (N) – It is defined as that much force which produces an acceleration of $1 m s^{-2}$ in a mass of one kilogram.

- Characteristics of a force –

1. Magnitude
2. Direction
3. Sense
4. Point of application.

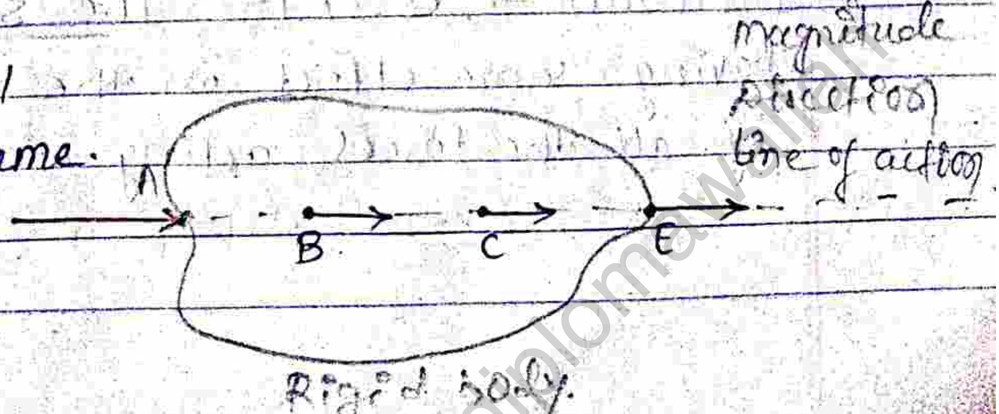



Effect of force –

- It may bring change in the motion of the body.
- It may bring all other force which are already acting on a body, in balance thus bringing the body to a state of rest or of equilibrium.

* Principal of transmissibility of force –
if the force acting on a rigid body at a point then, it can be transferred to the other points on the same body.

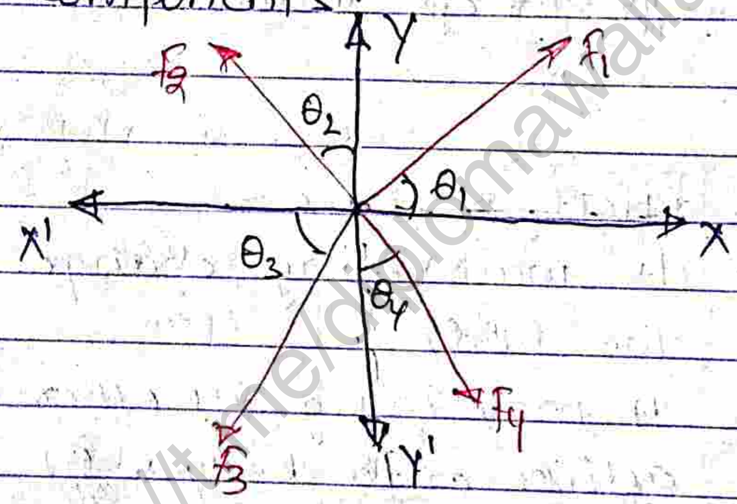
- Effect will remain same.

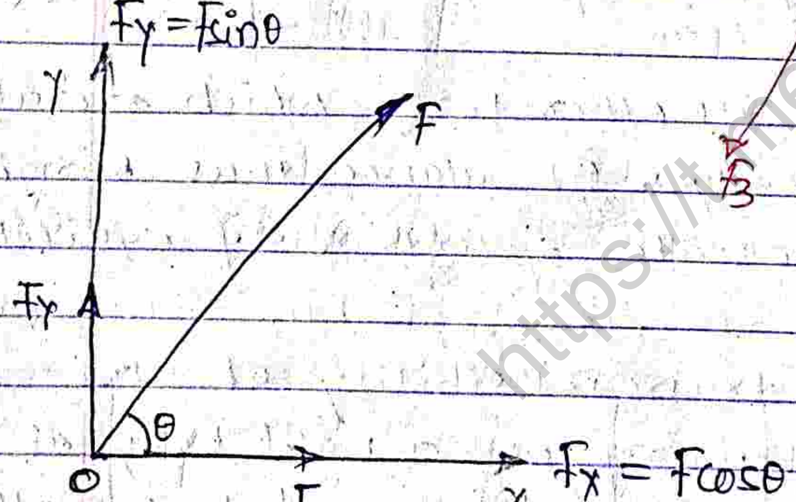


## UNIT-1
## Chapter-3 — Resolution of force

**Resolution of force** – The process of resolving / splitting the given concurrent forces into number of component without changing its effect on a body.

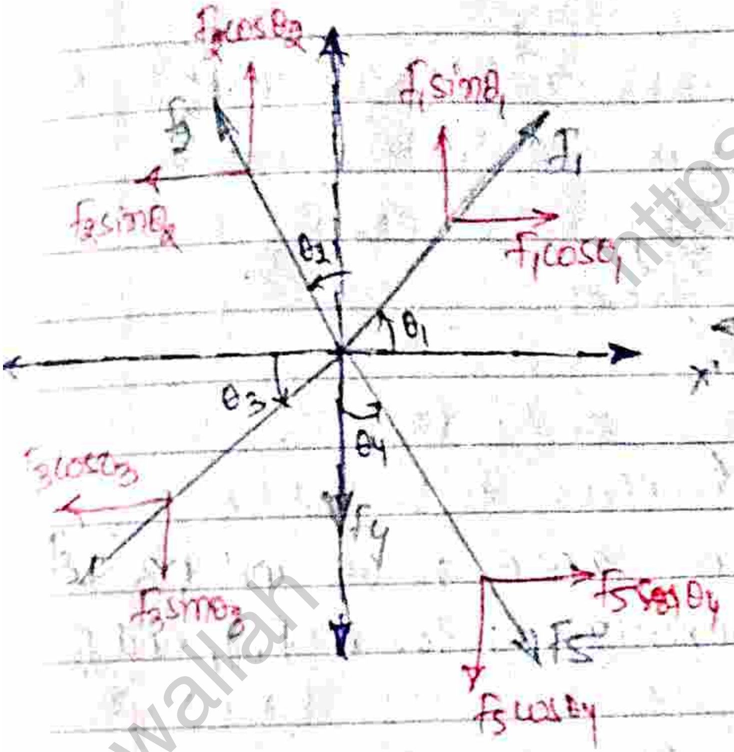
- force is resolved in two mutually perpendicular direction or into two rectangular components.

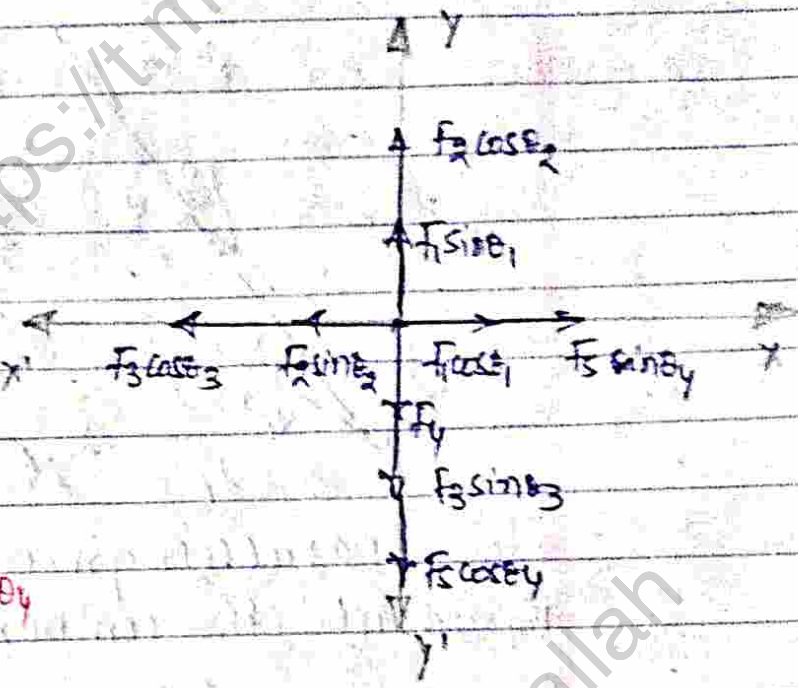



**Resultant force (R)** – The single force of having same effect on the body as that of all the forces acting on the same body.

## Mutually perpendicular components

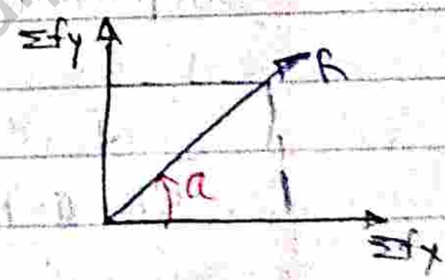



$$\Sigma F_x = F_1\cos\theta_1 + F_5\sin\theta_4 - F_2\sin\theta_2 - F_3\cos\theta_3$$

$$\Sigma F_y = F_2\cos\theta_2 + F_1\sin\theta_1 - F_4 - F_3\sin\theta_3 - F_5\cos\theta_4$$

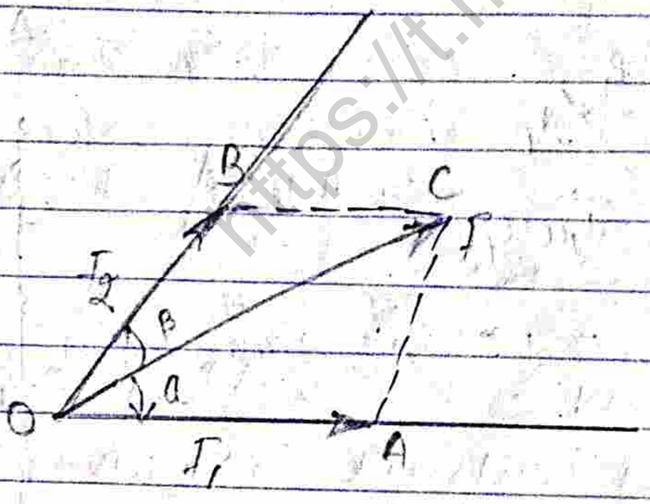
$$\boxed{R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}}$$ magnitude.




$$\tan a = \frac{\Sigma F_y}{\Sigma F_x}$$

$$\boxed{a = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)}$$ Direction.

# Non-perpendicular components




In parallelogram OACB ; OA and OB represent the components of force F, $F_1$ and $F_2$.

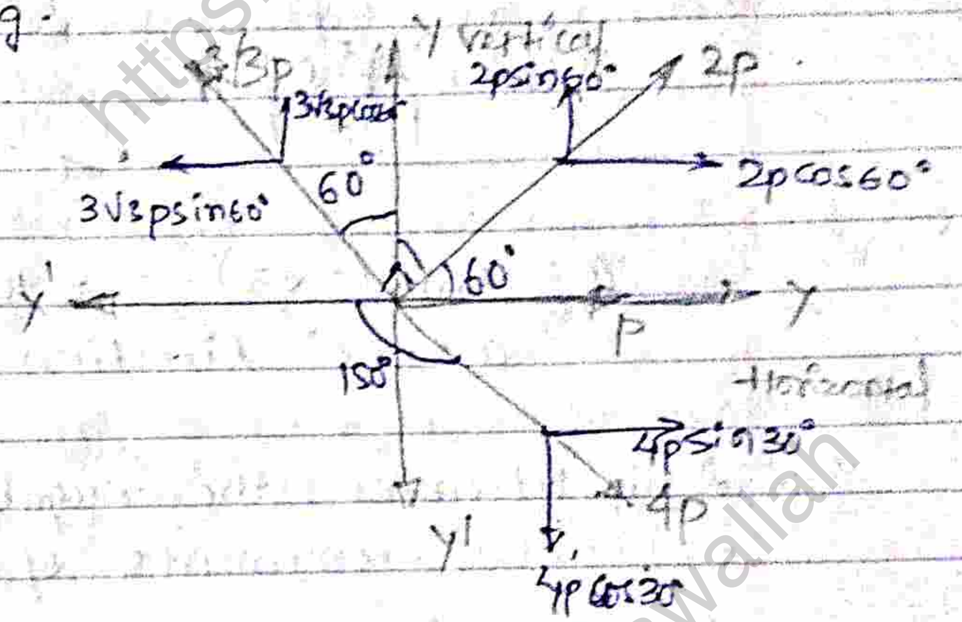
Applying sine rule in $\Delta$OAC.

$$\frac{OC}{\sin[\pi-(\alpha+\beta)]} = \frac{OA}{\sin\beta} = \frac{AC}{\sin\alpha}$$

$$\frac{F}{\sin(\alpha+\beta)} = \frac{F_1}{\sin\beta} = \frac{F_2}{\sin\alpha} \quad [\because AC = OB]$$

$$\boxed{F_1 = F\frac{\sin\beta}{\sin(\alpha+\beta)}} \; ; \; \boxed{F_2 = F\frac{\sin\alpha}{\sin(\alpha+\beta)}}$$

Q. find the magnitude and direction of resultant force R of four concurrent forces acting as shown in fig.




$$\Sigma f_x = 2p\cos60° + 4p\sin30° + p - 3\sqrt{3}p\sin60°$$

$$= 2p \times \frac{1}{2} + 4p \times \frac{1}{2} + p - 3\sqrt{3}p \times \frac{\sqrt{3}}{2}$$

$$= p + 2p + p - \frac{9p}{2} = 4p - \frac{9p}{2} = \frac{-p}{2}.$$

$$\Sigma f_y = 2p\sin60° + 3\sqrt{3}p\cos60° - 4p\cos30° \cancel{- 2p}$$

$$= 2p \times \frac{\sqrt{3}}{2} + 3\sqrt{3}p \times \frac{1}{2} - 4p \times \frac{\sqrt{3}}{2}$$

$$= \sqrt{3}p + \frac{3\sqrt{3}p}{2} - 2\sqrt{3}p = \frac{\sqrt{3}}{2}p.$$

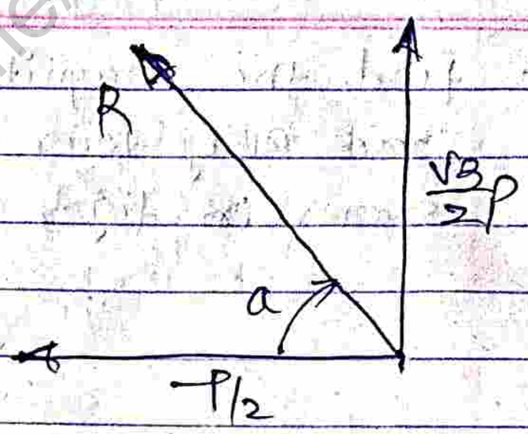
$$R = \sqrt{(\Sigma f_x)^2 + (\Sigma f_y)^2}$$

$$= \sqrt{\left(\frac{-p}{2}\right)^2 + \left(\frac{\sqrt{3}p}{2}\right)^2}$$

$$= \sqrt{\frac{p^2}{4} + \frac{3p^2}{4}} = \sqrt{p^2} = p.$$

$\therefore$ $\boxed{R = P}$ Magnitude.

$$\tan a = \frac{\Sigma f_y}{\Sigma f_x}$$

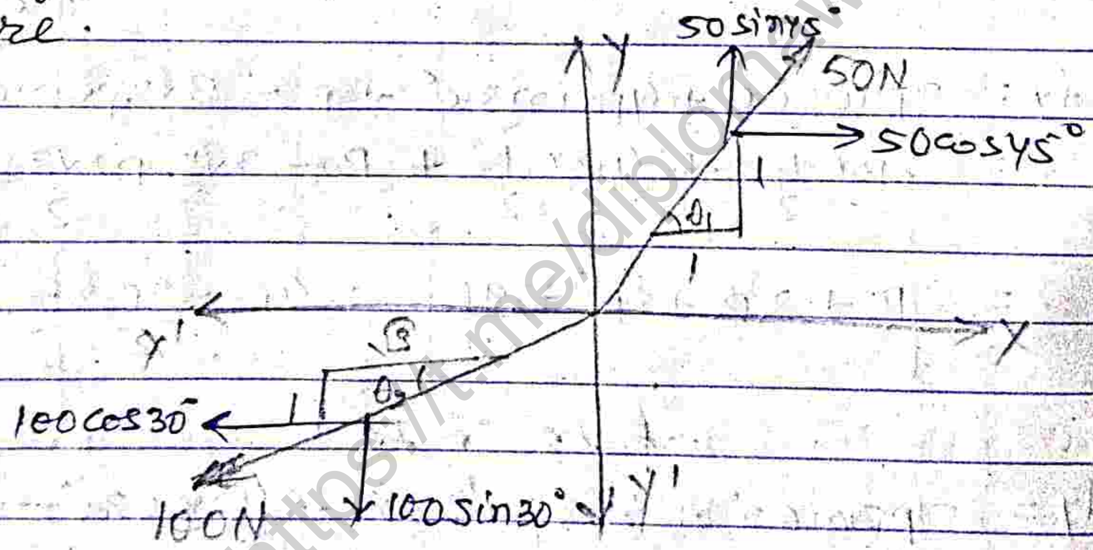



$$a = \tan^{-1}\left(\frac{-\frac{\sqrt{3}}{2}P}{\frac{P}{2}}\right)$$

$$a = \tan^{-1}(-\sqrt{3}) = -\tan^{-1}(\sqrt{3})$$

$a = -60^\circ$ direction with x-axis in c.w

Q. ~~A To~~ Determine the magnitudes of the rectangular components of forces shown in figure.




Sol^n :- $\theta_1 = \tan^{-1}(1) = 45^\circ$ ; $\theta_2 = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$

Magnitude of forces along x-axis.

$f_1 = 50\cos 45^\circ = 35.36N.$

$f_1 = 50\sin 45^\circ = 35.36N.$

$f_2 = -100\cos 30^\circ = -50\sqrt{3}\,N$

$f_2 = -100\sin 30^\circ = -50N.$

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Engg. Mechanics

UNIT-1
chapter-4

## Moment of forces

**Moment of force** — The tendency of force to rotate the body about any point or axis.

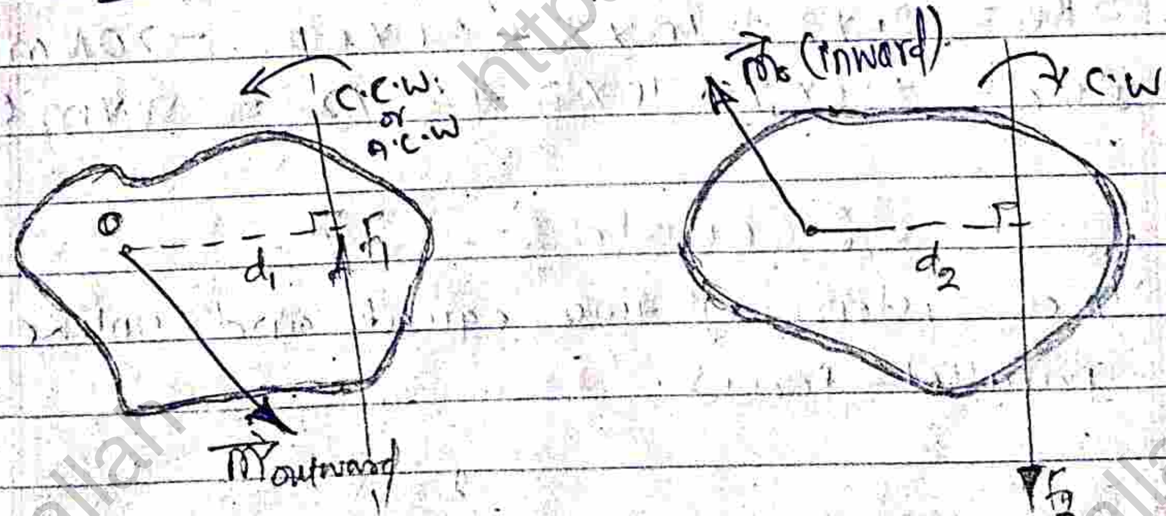
- **Measurement of moment of a force** — The product of magnitude of force and perpendicular distance from line of action of force to the point or axis about which we have to calculate the moment of force.

$$\boxed{M = F \times \perp d}$$

- It is vector physical quantity.

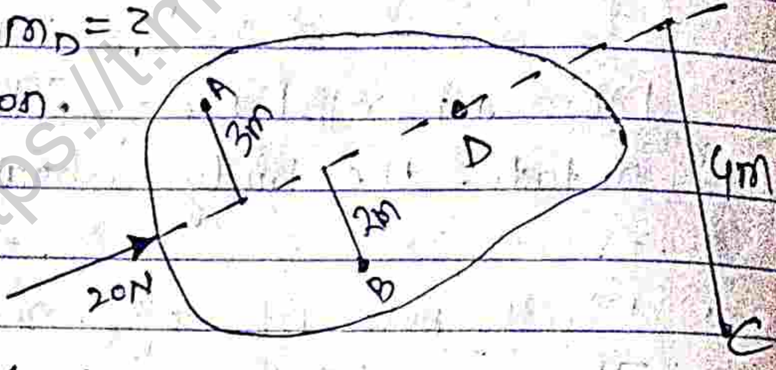
- Need parameter of moment of force —
→ Magnitude → Direction → sense of rotation.

(i) S.I unit — N-m.




- clock wise → Negative (−)
- Anticlock wise or counterclock wise → positive (+)

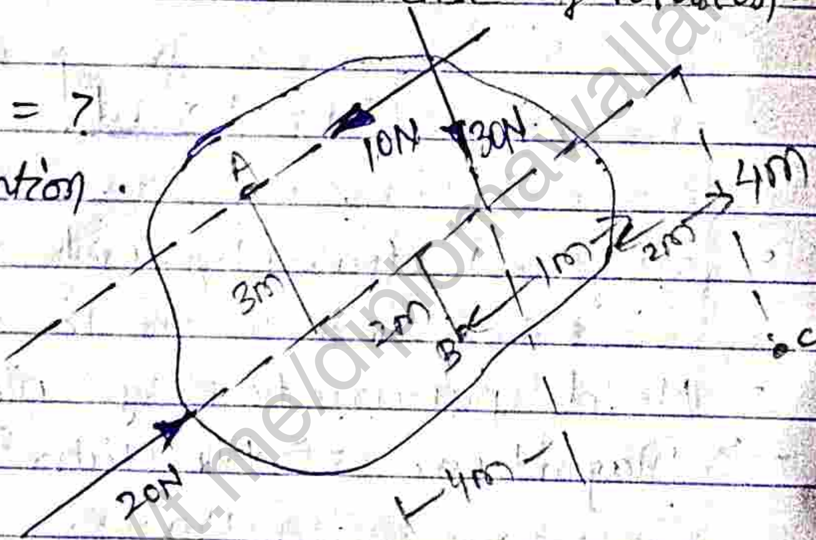
1. $M_A, M_B, M_C, M_D = ?$
& sense of rotation.




Sol$^n$:– $M_A = F \times d$

$M_A = 20 \times 3 = 60\,Nm$ c.c.w
$M_B = 20 \times 2 = -40\,Nm$ c.w
$M_C = 20 \times 4 = -80\,Nm$ c.w
$M_D = 20 \times 0 = 0$. No sense of rotation.

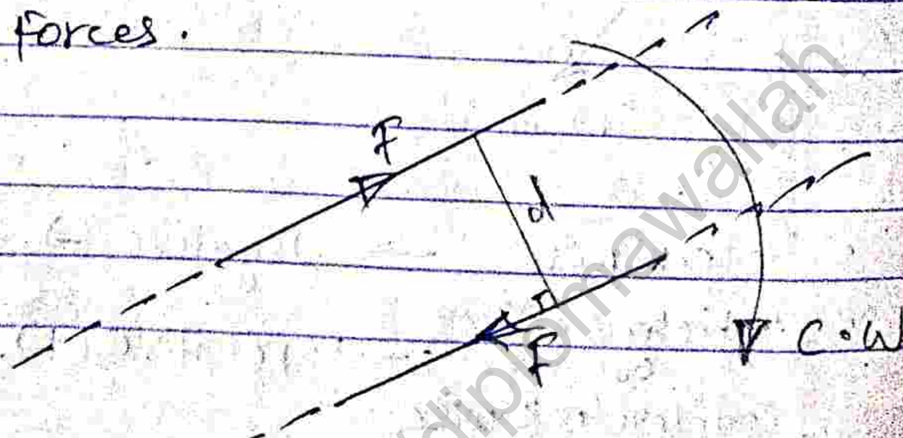
2. $M_A, M_B, M_C = ?$
& sense of rotation.




$\Sigma M_A = 10 \times 0 + 20 \times 3 - 30 \times 4 = -60N$ c.w
$\Sigma M_B = -20 \times 2 + 10 \times 5 - 30 \times 1 = -20\,Nm$ c.w
$\Sigma M_C = -20 \times 4 + 10 \times 7 + 30 \times 2 = 50\,Nm$ c.c.w

## Couple :–

The system of two equal and unlike parallel forces.



Moment of couple :- Two Equal and unlike parallel forces acting on a body and seperated by a distance a part, then moment of couple is the product of magnitude of force and perpendicular distance between line of action of forces.

$$\boxed{\text{Magnitude of couple or Moment of couple} = F \cdot d}$$

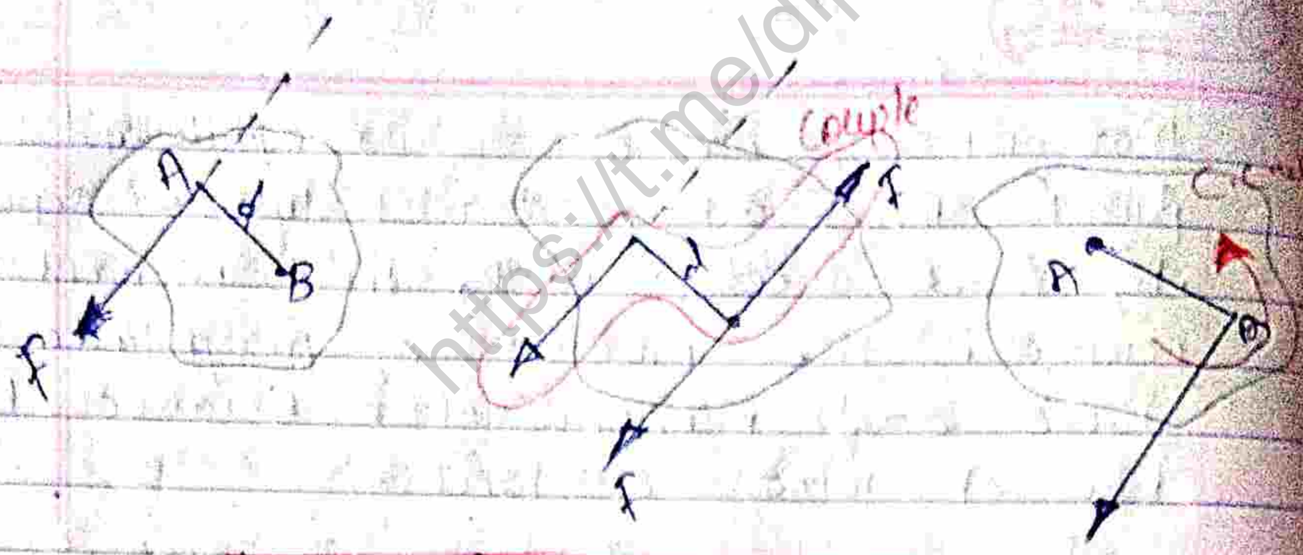
Examples of a couples:-

1. locking / unlocking of a lock with a key.
2. Winding of a watch or a clock.
3. Unscrewing the cap of an ink bottle.
4. Turn of the cap of a pen.
5. opening or closing a water tap.

## -: Force - couple system :-

→ Resolving a given force into a force and a couple.

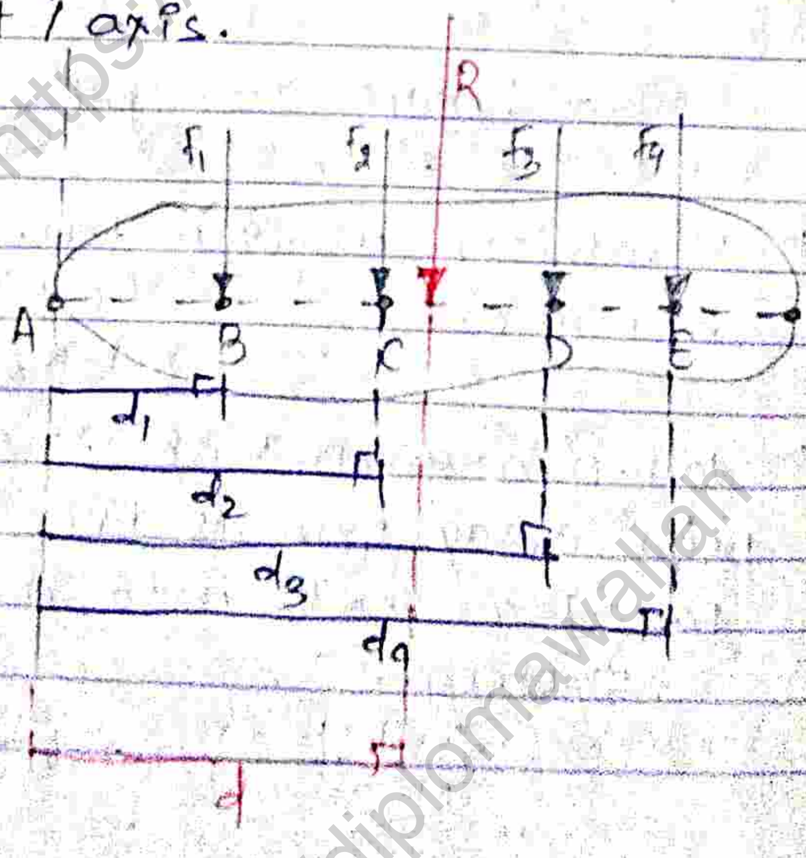
→ When a given force is transferred to another point away from its line of action, it will form / convert into a force - couple system.




## VARIGNON'S THEOREM

- If many forces are acting on the body, then net moment due to all forces about any point / axis will be equal to the moment due to resultant force alone about some point / axis.

- Used to find the location of point of application of resultant force w.r.t any point / axis.



Take moment of $F_1, F_2, F_3, \& F_4$ about point 'A'.

$$\Sigma M_A = -F_1 d_1 - F_2 d_2 - F_3 d_3 - F_4 d_4$$

$$\Sigma M_A = -(F_1 d_1 + F_2 d_2 + F_3 d_3 + F_4 d_4) \quad \text{---} \; (i)$$

Take moment of R about point A

$$M_R = -R \times d \quad \text{---} \; (ii)$$

According to V. theorem:

$$\Sigma M_A = M_R$$

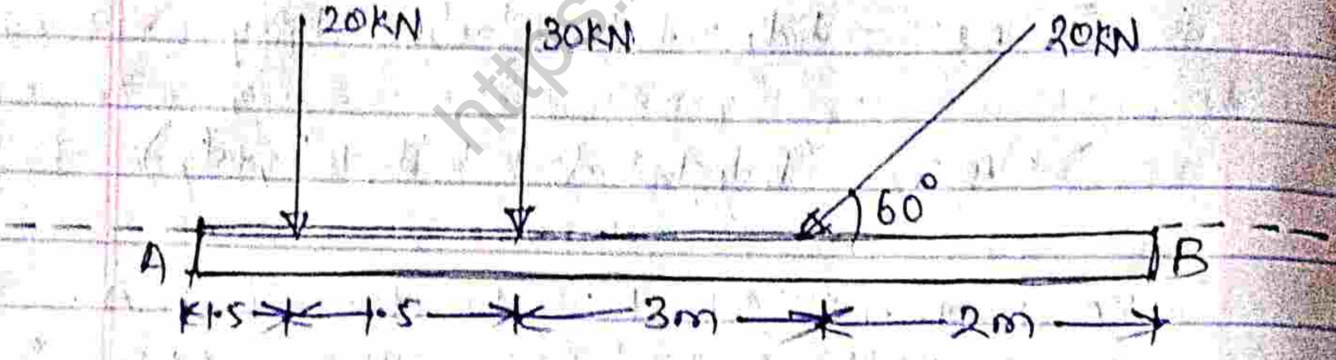
$$-(F_1 d_1 + F_2 d_2 + F_3 d_3 + F_4 d_4) = -R \times d$$

$$\boxed{R \times d = F_1 d_1 + F_2 d_2 + F_3 d_3 + F_4 d_4}$$

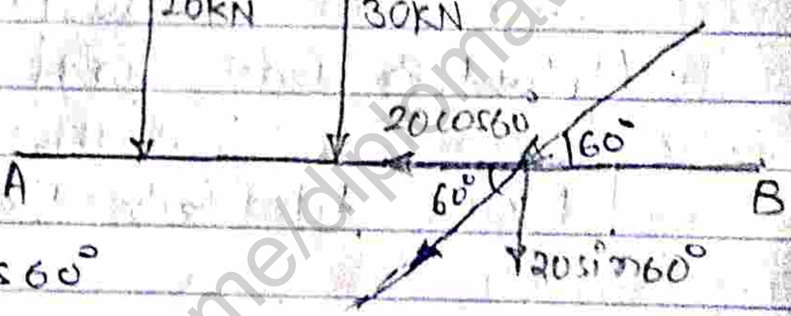
- Resultant of coplanar:

1. Magnitude — $\boxed{(R) = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}}$
2. Direction — $\boxed{a = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)}$
3. location — $\boxed{d = \frac{M_R}{R}}$

Q. The system of forces acting on a beam AB as shown.




Determine the magnitude, direction & location of point of application of resultant force of given force system w.r.t point A.




Sol$^n$:- • Magnitude –

$\Sigma f_x = -20\cos 60°$

$= -20 \times \frac{1}{2} = -10 KN.$

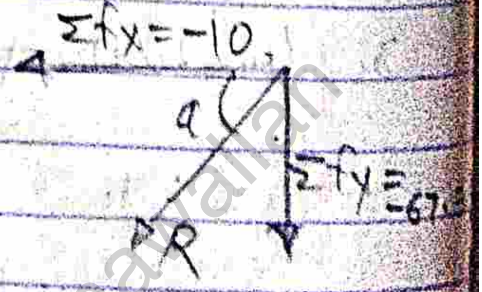
$\Sigma f_y = -20 - 30 - 20\sin 60°$

$= -50 - 20 \times \frac{\sqrt{3}}{2} = -67.32 KN$

$R = \sqrt{(\Sigma f_x)^2 + (\Sigma f_y)^2}$

$R = \sqrt{(-10)^2 + (-67.32)^2}$

$R = 68.05 KN.$




• Direction –

$a = \tan^{-1}\left(\frac{\Sigma f_y}{\Sigma f_x}\right) = \tan^{-1}\left(\frac{-67.32}{-10}\right) = 81.55°$

- location of point of application.

$\Sigma M_A = -f_1d_1 - f_2d_2 - f_3d_3$

$= -(20 \times 1.5 + 30 \times 3 + 20 \times 6 \times \sin 60^\circ)$

$= -(3 + 90 + 120\frac{\sqrt{3}}{2})$

$= -(3 + 90 + 60\sqrt{3})$

$M_R = -R\sin\alpha \times d$

$= -68.05 \times \sin 81.55 \times d$.

According to V. theorem –

$\Sigma M_A = M_R$

$\cancel{-}(3 + 90 + 60\sqrt{3}) = \cancel{-} 68.05 \times \sin 81.55^\circ \times d$

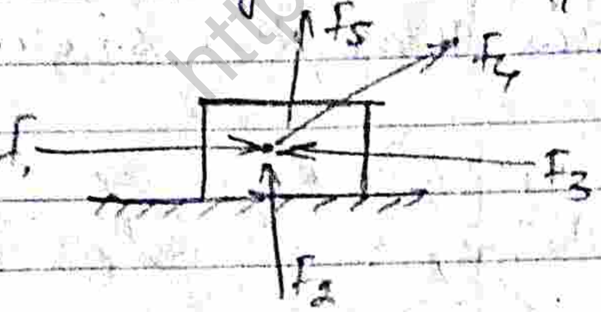
$d = \dfrac{3 + 90 + 60\sqrt{3}}{68.05 \times \sin(81.55^\circ)} = 3.33m.$

$\boxed{d = 3.33m}$ location of point of application of R w.r.t A.

UNIT-1

chapter-5 **Composition of forces**

Force system — A group of two or more than two forces acting on a body at a particular instant.

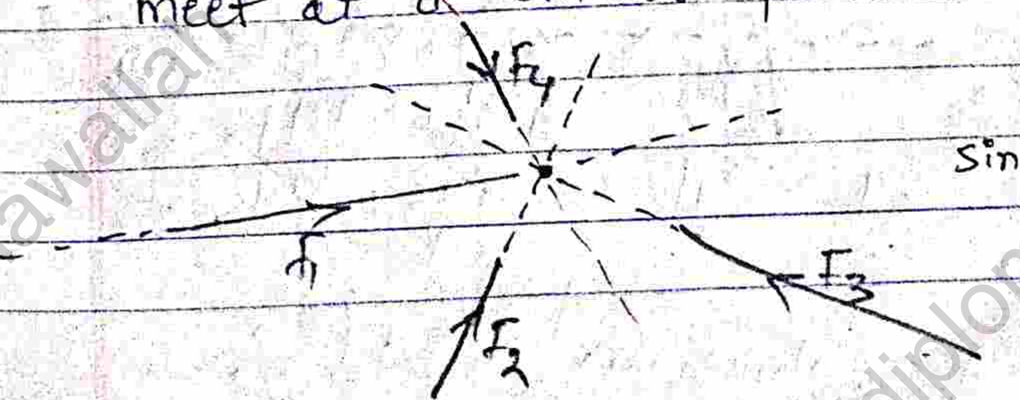



## Types of force system

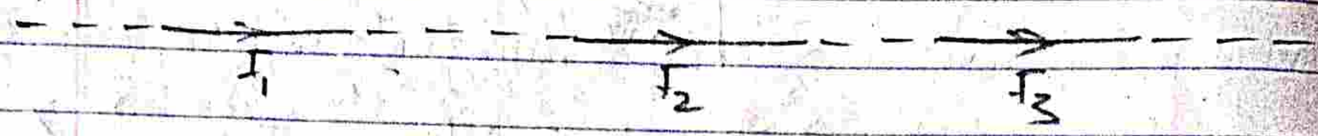
- parallel force system
- concurrent force system
- collinear force system

I. parallel force system — if the line of action of all forces are parallel to each other.

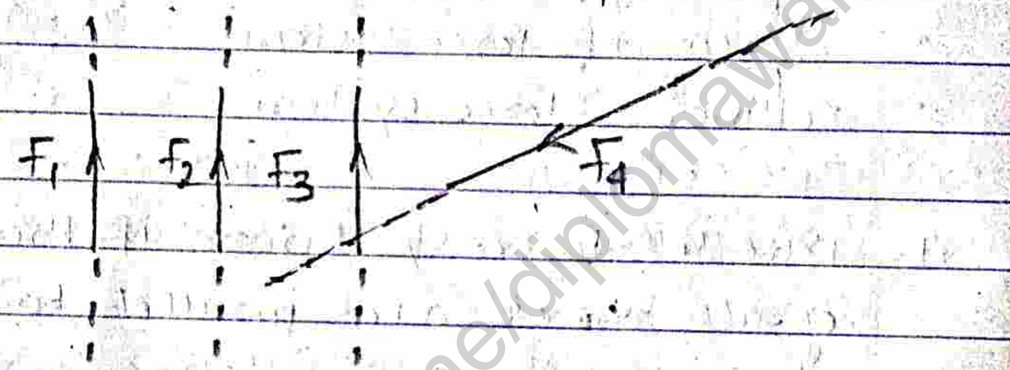

II. Concurrent force system — if the line of action of all the forces in the system are passes or meet at a common point.



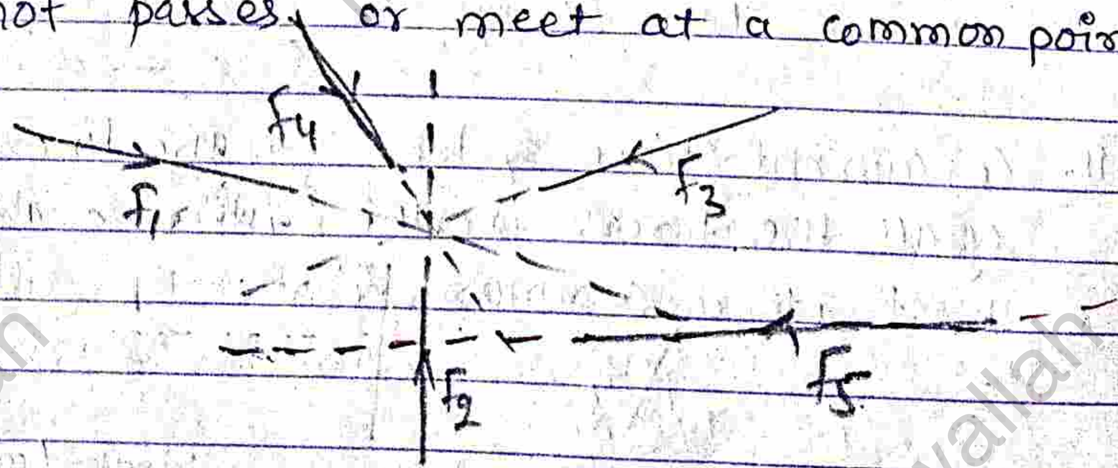
III. Collinear force system — if the line of action of all the forces in the system are passes through a single line.



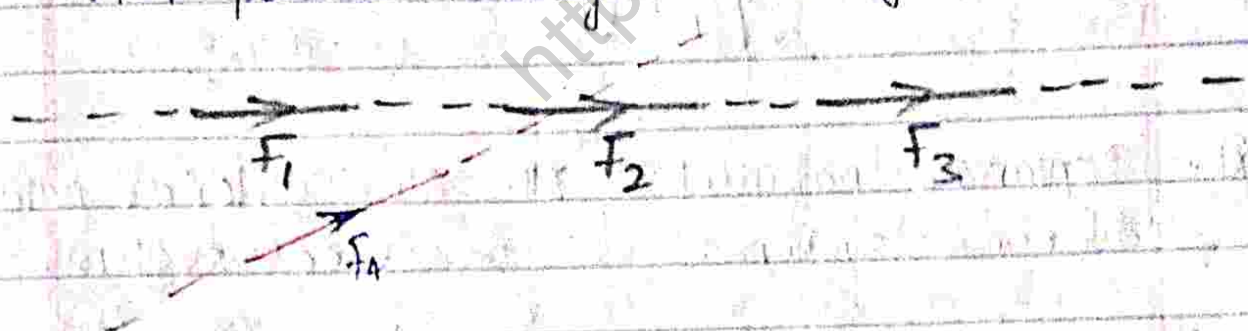

IV. Non-parallel force system — if the line of action of all forces are not parallel to each other.



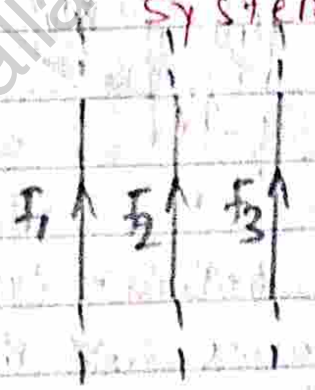

V. Non-Concurrent force system — if the line of action of all forces in the system are not passes or meet at a common point.



VI. Non-collinear force system — if the line of action of all the forces in the system are not passes through a single line.

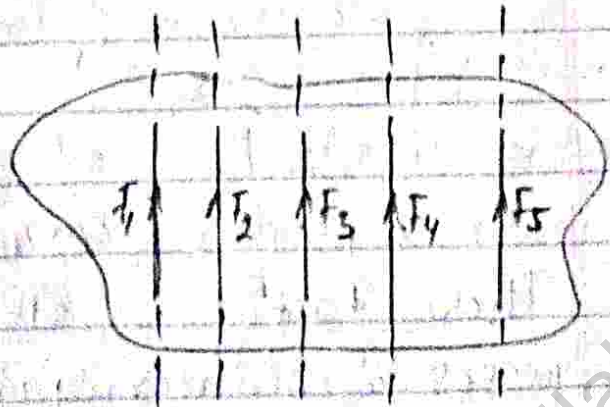



VII. Like parallel force system.

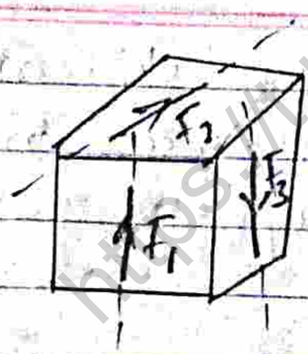



VIII. Unlike parallel force system.

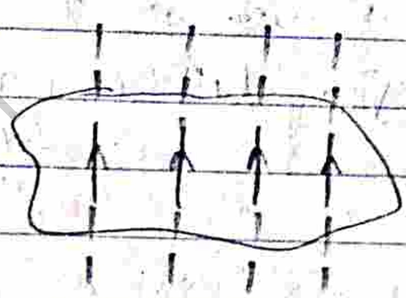
IX. Coplanar forces — if the lines of action of all the forces lie on the same plane, the system is called coplanar forces system.



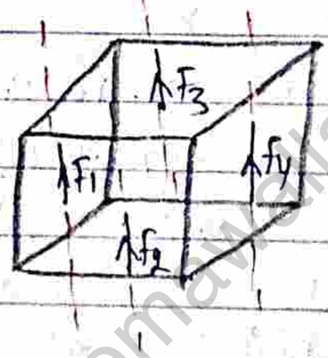

X. Non-coplanar force system — if the lines of action of all the forces lie on the different planes, the system is called non-coplanar force system.



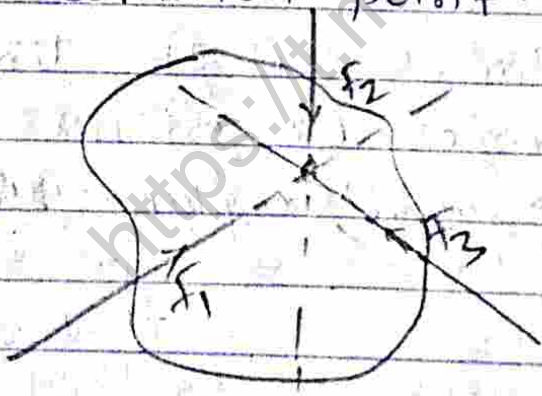

XI. Coplanar parallel forces system.



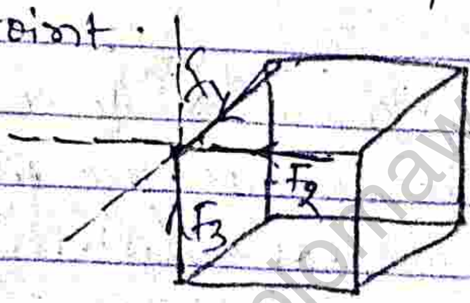
XII. Non-coplanar parallel forces system.




XIII. Coplanar concurrent force system - These forces exist in same planes and its posses a common point.

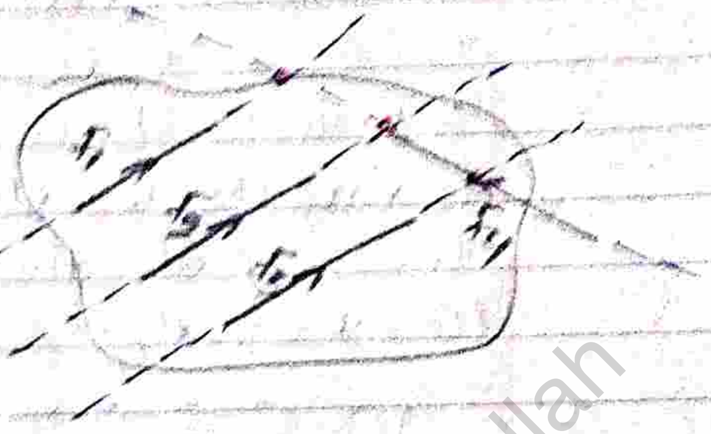



XIV. Non-coplanar concurrent force system - These forces exist in different planes but posses a common point.



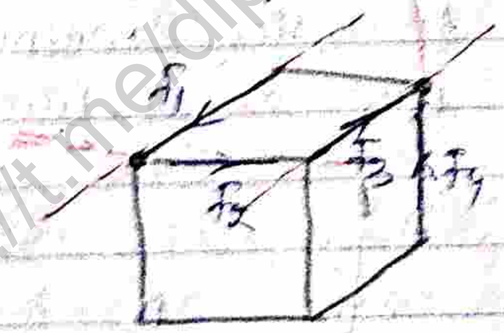
ii) Coplanar Non-Concurrent force system —

These are forces act in same planes and they posses do not one single point of concurrency.



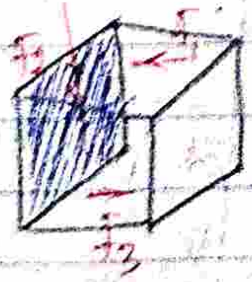

iii) Non-coplanar Non-concurrent forces system —

These forces act in different planes and they do not posses one single point of concurrency.




iv) Coplanar collinear force system :-


Non-coplanar Non-collinear force system :




• Non coplanar collinear force system are not possible.

## Composition of force

Two system of forces are said to be equivalent if they produce the same mechanical effect on a rigid body.

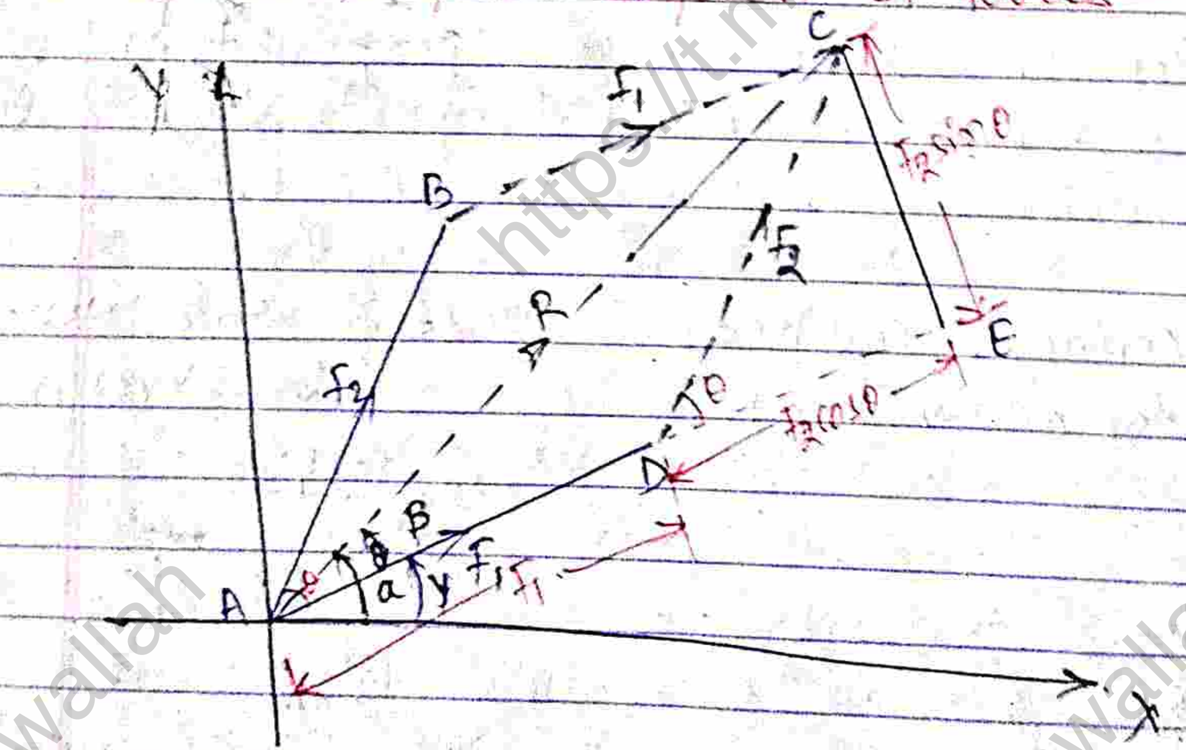
**Resultant force (R)** — A single force of having same effect on the body as that of all the forces acting on the same body.

### Methods of composition of forces

- Analytical methods / Trigonometric method.
- Graphical methods.

## I. Analytical methods —

- **Law of parallelogram of forces** —



from $\Delta ACE$

$$AC^2 = CE^2 + AE^2$$

$$R^2 = (f_2 \sin\theta)^2 + (f_1 + f_2\cos\theta)^2$$

$$= f_2^2 \sin^2\theta + f_1^2 + f_2^2\cos^2\theta + 2f_1f_2\cos\theta$$

$$= f_1^2 + f_2^2(\sin^2\theta + \cos^2\theta) + 2f_1f_2\cos\theta$$

$$= f_1^2 + f_2^2 + 2f_1f_2\cos\theta$$

$$\boxed{R^2 = f_1^2 + f_2^2 + 2f_1f_2\cos\theta}$$

$$\boxed{|R| = \sqrt{f_1^2 + f_2^2 + 2f_1f_2\cos\theta}}$$

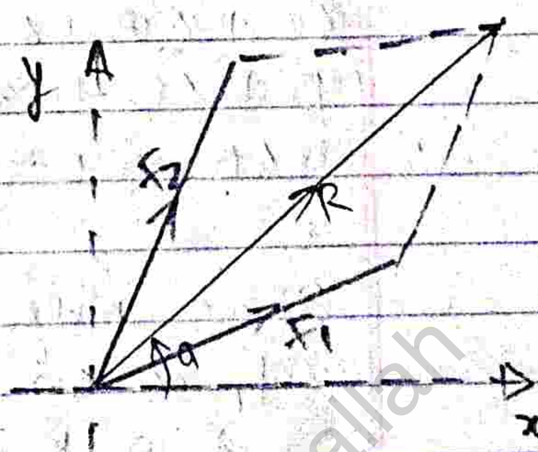
Again,

$\Delta ACE$

$$\boxed{\tan\beta = \frac{f_2\sin\theta}{f_1 + f_2\cos\theta}}$$

$$\boxed{\beta = \tan^{-1}\left(\frac{f_2\sin\theta}{f_1 + f_2\cos\theta}\right)}$$

II. Graphical method –




$f_1 = 40N$

$f_2 = 60N$

$\theta = 30°$

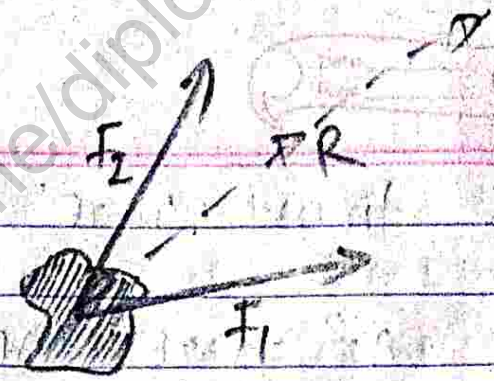
let $1N = 0.1cm$

$f_1 = 40 \times 0.1 = 4cm$

$f_2 = 60 \times 0.1 = 6cm$.

$\therefore$ $R = 4.5cm$ then, $R = \frac{4.5}{0.1}N = 45N$.

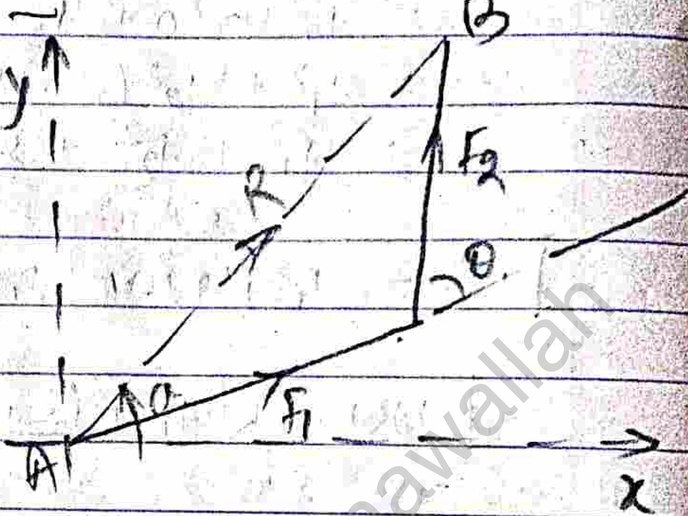
- law of triangle —




I. Graphical method —

$f_1 = 40N$

$f_2 = 60N$.




let, $1N = 0.1cm$.

$f_1 = 40N \times 0.1 = 4cm$

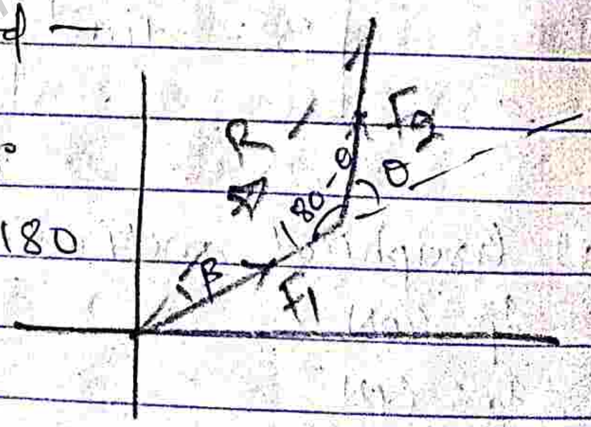
$f_2 = 60N \times 0.1 = 6cm$.

$\theta = 30^\circ$

$R = AB = 4.5cm$.

then, $R = \frac{4.5cm}{0.1N} = 45N$.

II Analytical method —




$\angle A + \angle B + \angle C = 180^\circ$

$\beta + \angle B + 180 - \theta = 180$

$\angle B = \theta - \beta$

Sine rule apply —

$f_1 \propto \sin(\theta - \beta)$

$f_1 = K\sin(\theta - \beta)$

$\frac{f_1}{\sin(\theta-\beta)} = K$ - - - - (i)

$\sin(180-\theta)$

Q.1) if the maximum and minimum resultant force of two forces acting at a point on a rigid body are 40KN and 10KN respectively then what will be the value of two forces.

Sol$^n$:- $R_{max} = 40KN$ ; $R_{min} = 10KN$

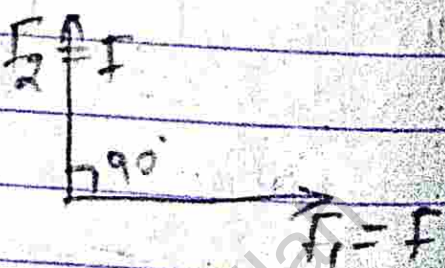
$$R_{max} = f_1 + f_2 = 40KN$$
$$R_{min} = f_1 - f_2 = 10KN$$

$$\begin{aligned} f_1 + \cancel{f_2} &= 40 \\ f_1 - \cancel{f_2} &= 10 \\ \hline 2f_1 &= 50 \\ \boxed{f_1 = 25} \end{aligned}$$

$$\begin{aligned} f_1 - f_2 &= 10 \\ 25 - f_2 &= 10 \\ -f_2 &= 10 - 25 \\ \boxed{f_2 = 15KN} \end{aligned}$$

Q.2) if two tensile forces each of magnitude F are acting at a point on body perpendicular to each other then their resultant force will be -- (I) $\sqrt{2}F$ (II) 0 (III) $\sqrt{2}$ (IV) $\sqrt{2}f$.

$$|R| = \sqrt{f_1^2 + f_2^2 + 2f_1f_2\cos\theta}$$
$$= \sqrt{F^2 + F^2 + 2FF\cos 90^\circ}$$
$$= \sqrt{2F^2} = \sqrt{2}F$$



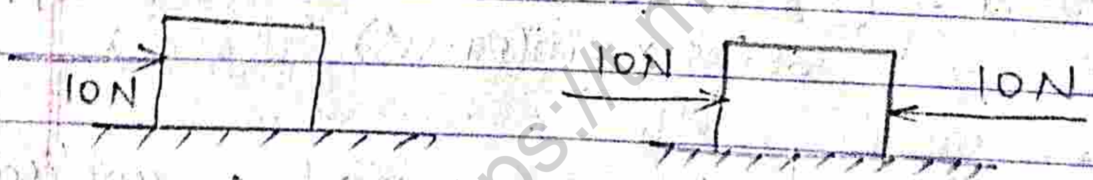
# UNIT-2 Equilibrium

## Equilibrium

Everybody continous to be in a state of rest or uniform motion in a straight line except in so far as it may be compelled by force to change that state.

- Equilibrium of forces – The forces acting on body are balanced, then forces are in equilibrium.

- Equilibrium of body – Body having zero effect under the action of applied forces, then body will be in equilibrium & applied forces are also in equilibrium.




Body in Non-Equilibrium.    Body in Equilibrium.

- Active force – External force that they to move the body.

- Reactive force – force that try to oppose the body.

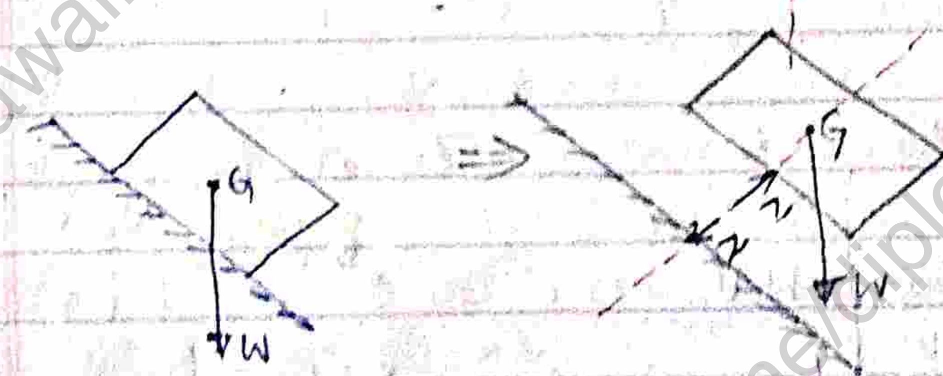
e.g → Normal force / reaction force, support reaction, Tension in string / rope ... etc.

## Free Body Diagram (FBD) :—

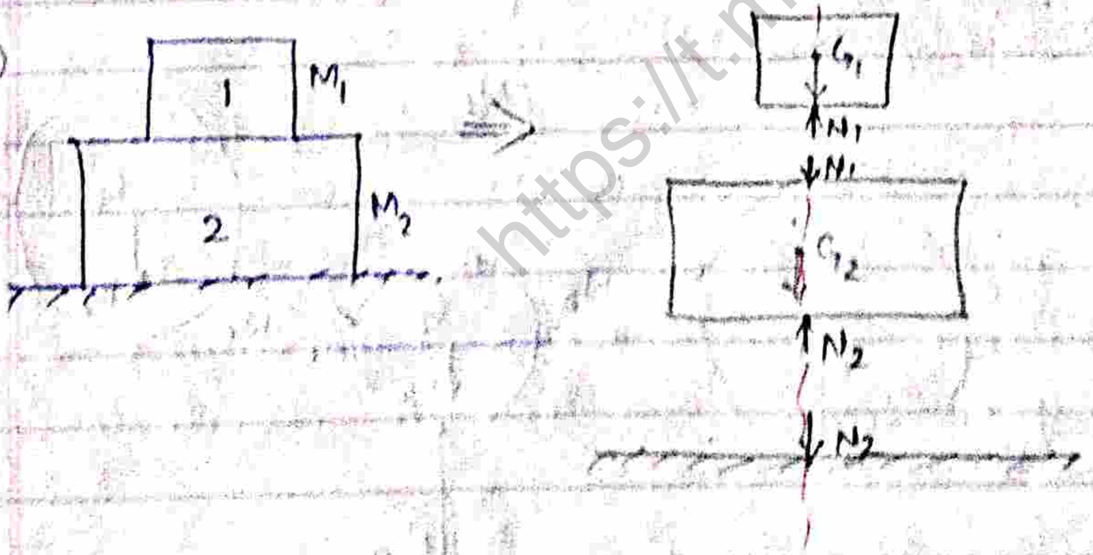
FBD is the diagram of the body after seprating all supports (wall, floor) ..etc and all contacts and represents all active and reactive forces applied on it. if the body is in Equilibrium.

①


②

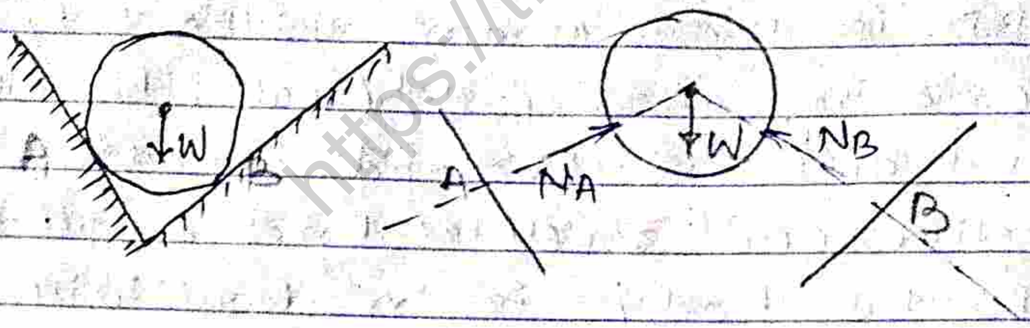



③

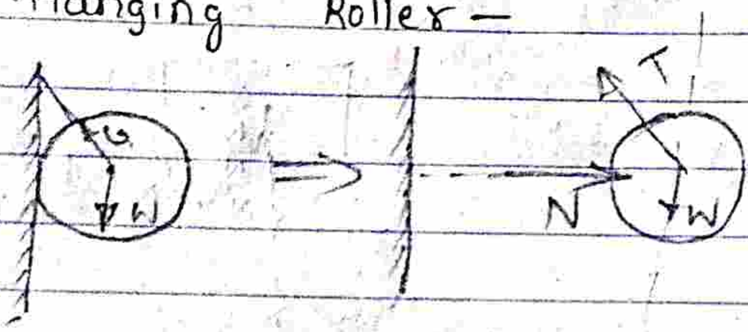



④ Sphere / ball resting on Surface —

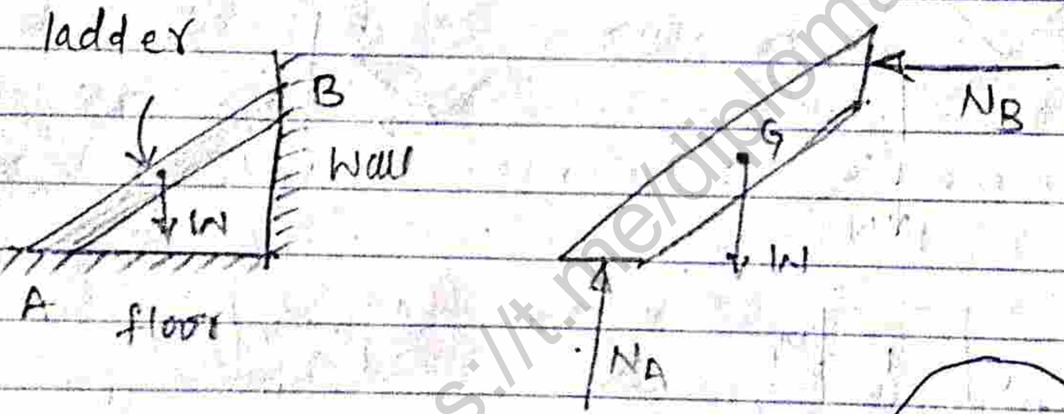
(5) sphere in V-groove –



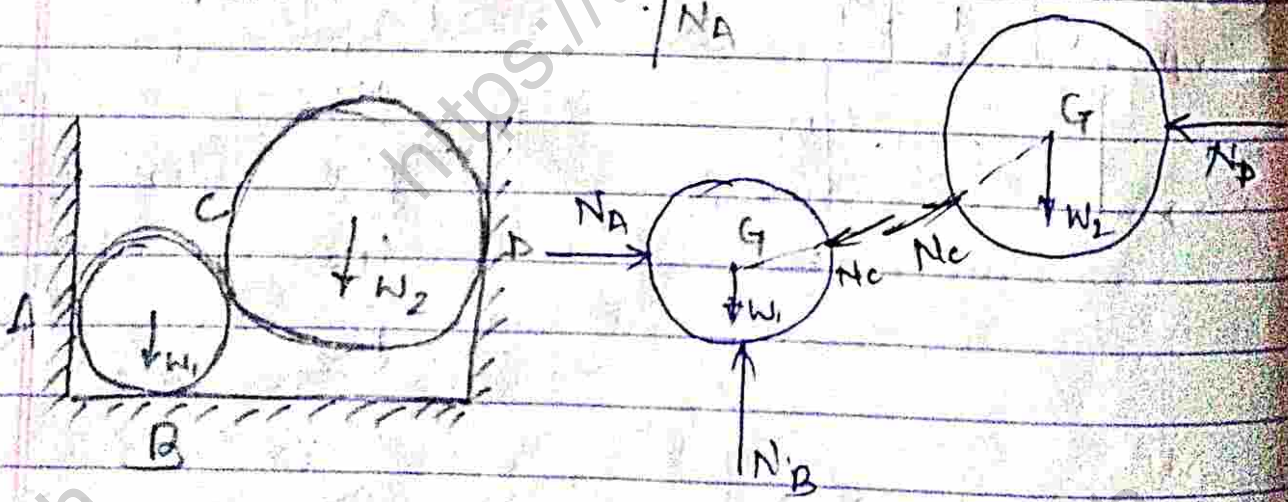

(6) Hanging Roller –



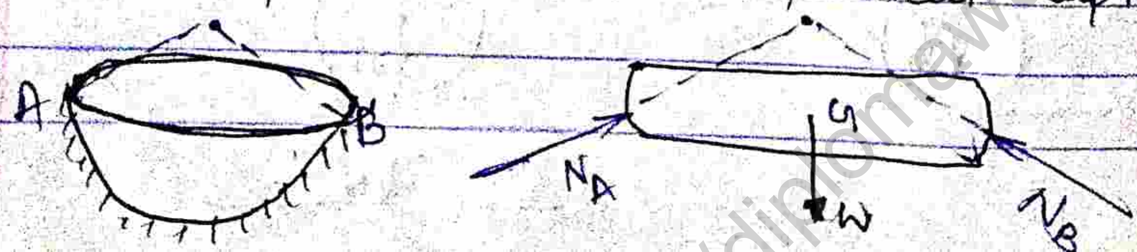

(7) ladder




(8)




(9) A ball placed in a hemispherical cup.



# Type of supports

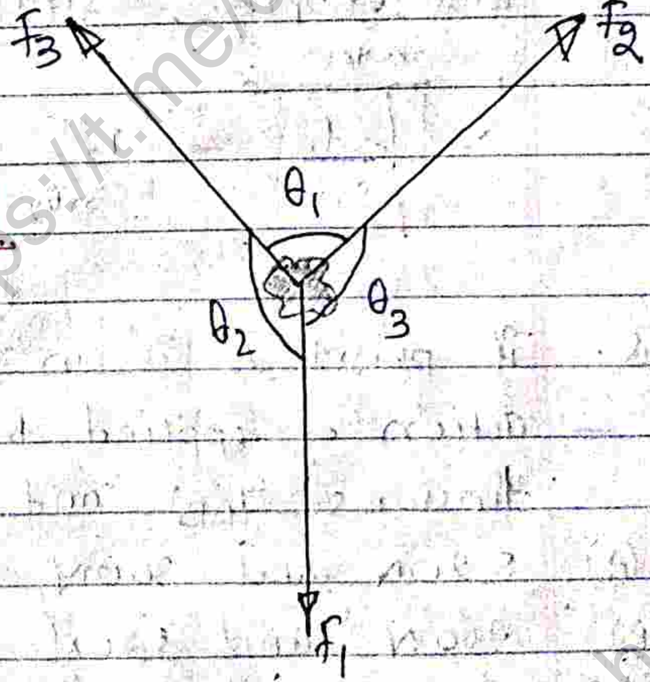
1. Roller support (1 - reactive force).

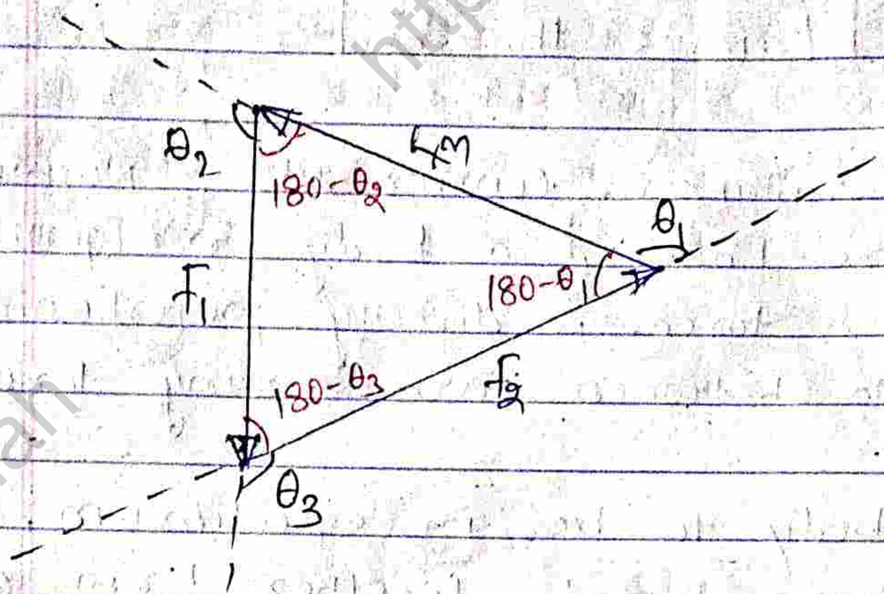
# Lami's theorem

if three coplanar, concurrent & Non-collinear forces are holding a body in Equilibrium then, Each force is directly proportional to sine angle between two remaing forces.

for the body to be in equilibrium, all these three forces together form a triangle in same order.

$$\boxed{\frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_3}{\sin\theta_3}}$$





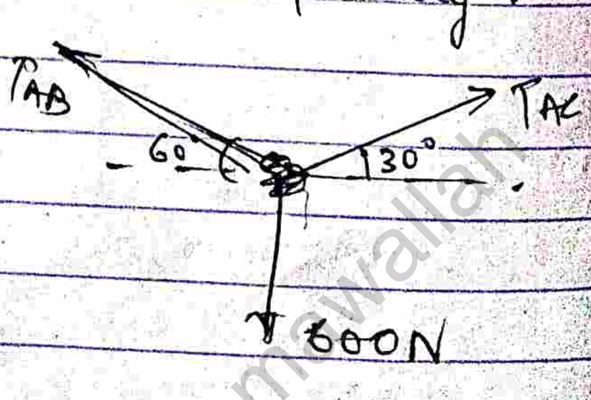

According to sine-rule —

$$\frac{f_1}{\sin(180-\theta_1)} = \frac{f_2}{\sin(180-\theta_2)} = \frac{f_3}{\sin(180-\theta_3)}$$

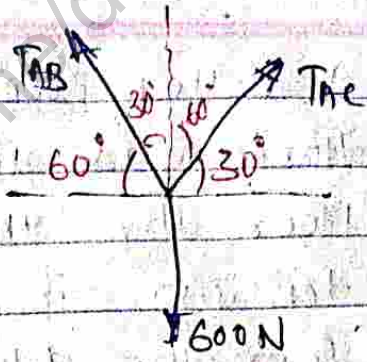
$$\boxed{\frac{f_1}{\sin\theta_1} = \frac{f_2}{\sin\theta_2} = \frac{f_3}{\sin\theta_3}}$$ Lami's theorem.

Q. If point A is in equilibrium under the action of applied forces, the values of tensions $T_{AB}$ and $T_{AC}$ are respectively.

(A) 520N and 300N
(B) 300 N and 520N
(C) 450N and 150N
(D) 150N and 450N.



Sol$^n$ :-




$$\frac{f_1}{\sin\theta_1} = \frac{f_2}{\sin\theta_2} = \frac{f_3}{\sin\theta_3}$$

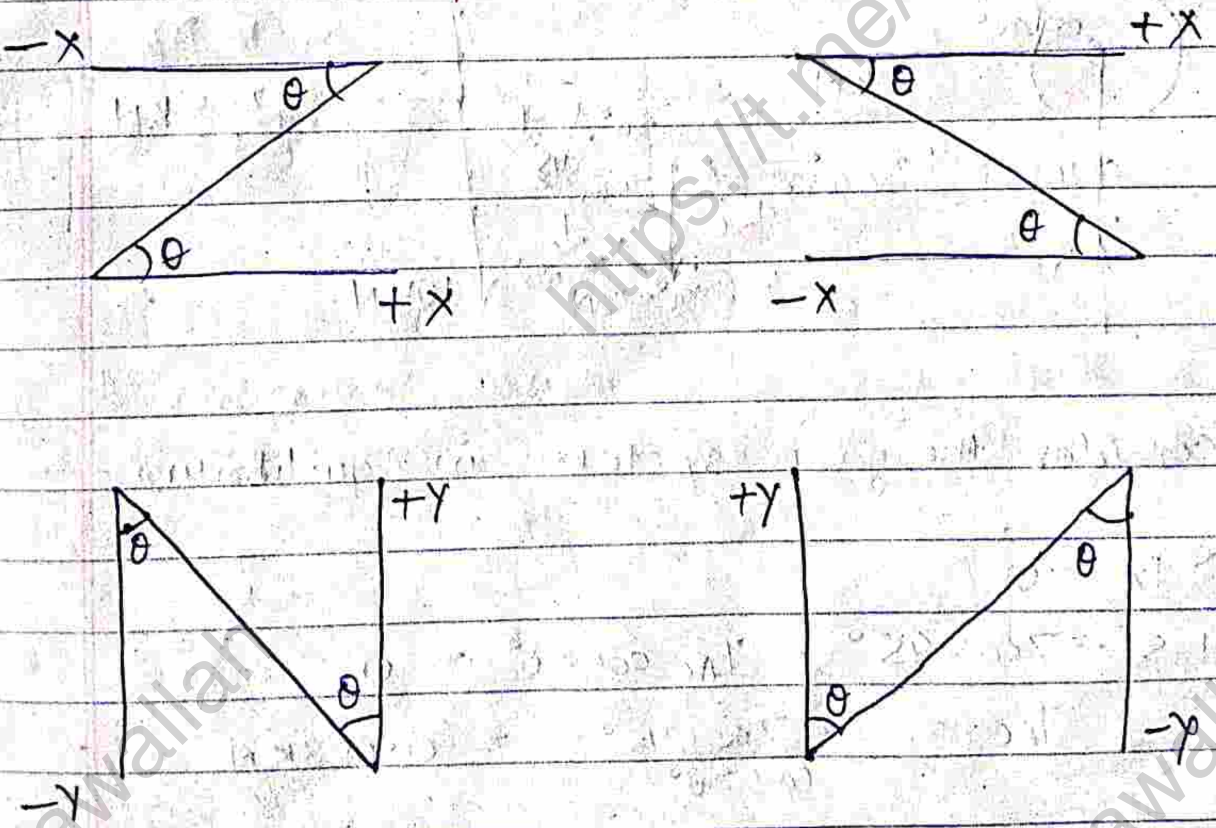
$$\frac{600}{\sin 90^\circ} = \frac{T_{AC}}{\sin 150^\circ} = \frac{T_{AB}}{\sin 120^\circ}$$

$$\frac{600}{1} = \frac{T_{AC}}{\frac{1}{2}} = \frac{T_{AB}}{\frac{\sqrt{3}}{2}}$$

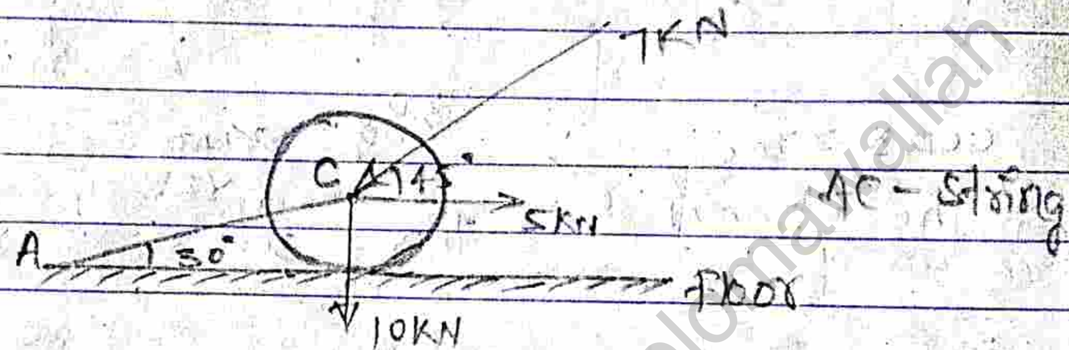
$600 = 2T_{AC}$ ; $\cancel{600}\ 300 = \dfrac{2T_{AB}}{\sqrt{3}}$

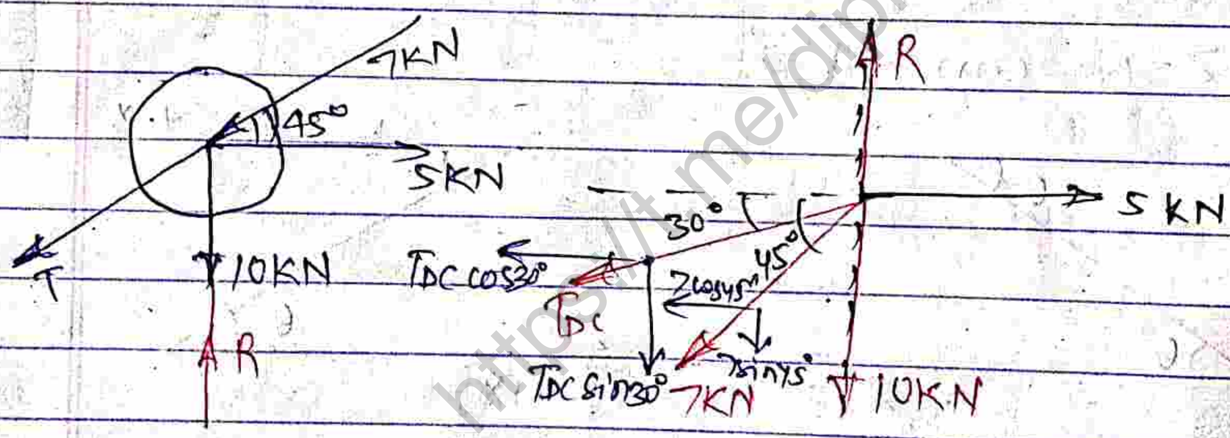
$T_{AC} = 300\,N$ ; $T_{AB} = 300\sqrt{3} = 300 \times 1.73 = 520\,N$

Z-N concept :—



Q. A roller weighing 10KN rests on a smooth horizontal floor and is connected to the floor by the string AC. Determine the tension in the string AC and reaction from the Floor, if the roll is subjected to the horizontal force of 5KN and an inclined force of 7KN as shown in fig.




consider the given system in Equilibrium —

$\Sigma f_x = 0$

$+5 - 7\cos 45° - T_{AC} \cos 30° = 0$

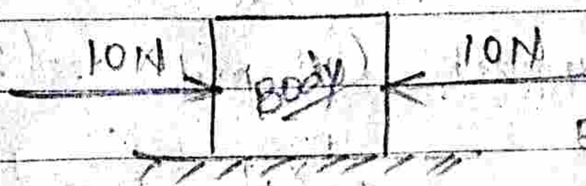
$$T_{AC} = \frac{5 - 7\cos 45°}{\cos 30°} = 0.058\,KN$$

$\Sigma f_y = 0$

$-10 - T_{AC}\sin 30° - 7\sin 45° + R = 0$

$R = 14.97\,KN$

Equilibrant → The force which is equal in magnitude, opposite in direction and collinear to the resultant of the force system & which is responsible to bring the body or particle in state of Equilibrium is called an Equilibrant.

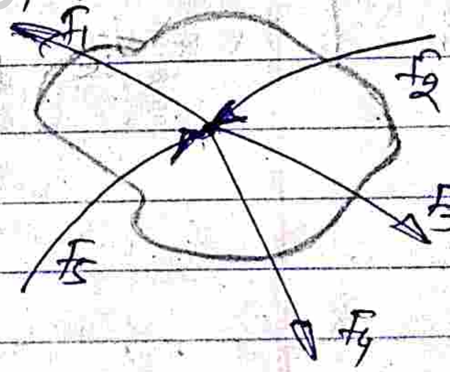



Body in Equilibrant

$\boxed{E = -R}$

- magnitude same as that of resultant.
- line of action same as resultant.
- Direction opposite ~~to~~ that of resultant.

* Coplannar concurrent Force system —




Net effect → Resultant Force.

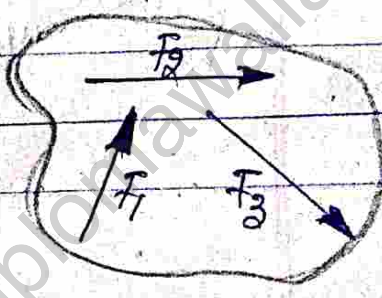
$R \rightarrow \Sigma f_x , \Sigma f_y$

Equilibrium condition ; $R = 0$

Static Equilibrium Equation.

$\Sigma f_x = 0 \; ; \; \Sigma f_y = 0$

* Coplanner Non-conurrent force system —




Equilibrium condition → $R = 0$

$\Sigma M_{any\ point} = 0$

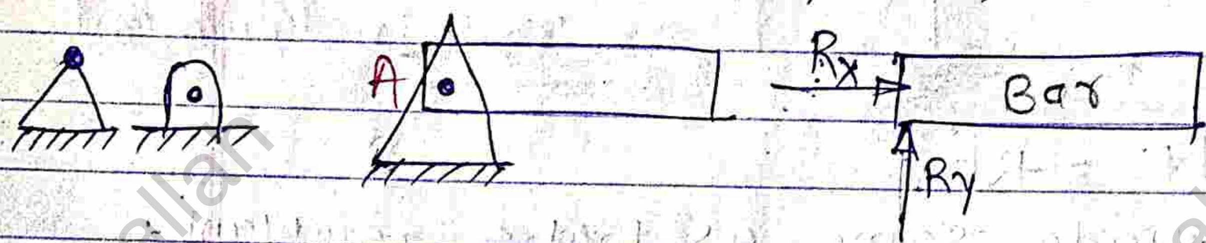
Static Equilibrium Equation —

$\Sigma f_x = 0 \; ; \; \Sigma f_y = 0 \; ; \; \Sigma M_{any\ point} = 0.$

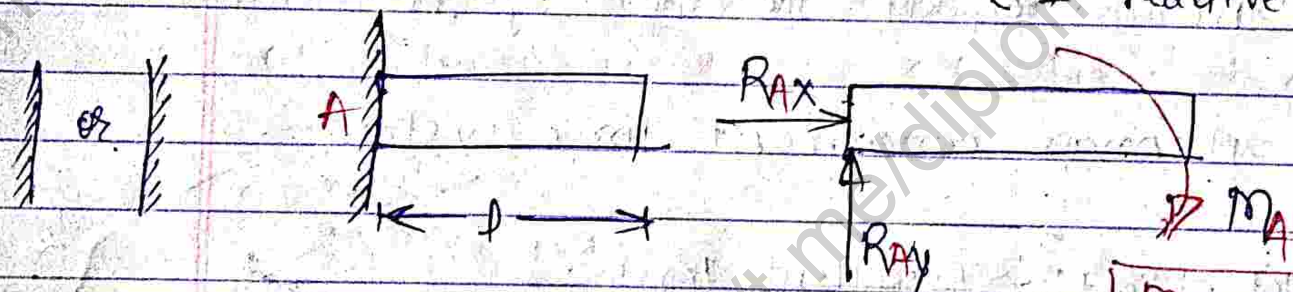
# Types of support

1. Roller support ( 1 - reactive force)


2. Hinged support (pinned) — (2-reactive force)



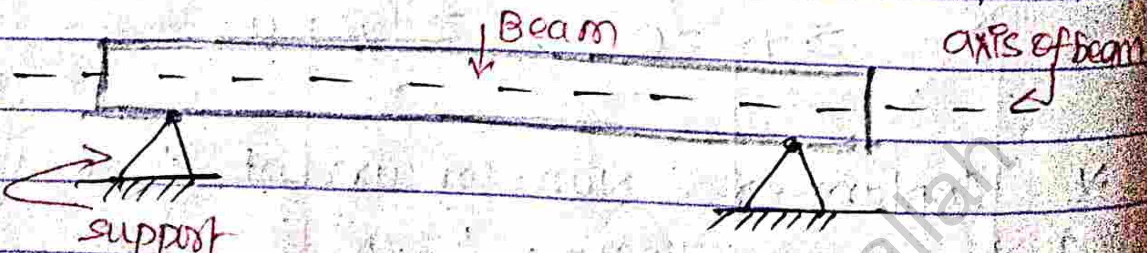

3. fixed / Built support (2 - reactive force, 1 - reactive moment)




$\boxed{M_p = W \times l}$

# BEAM

A horizontal structural member designed to resist forces transverse to its axis.

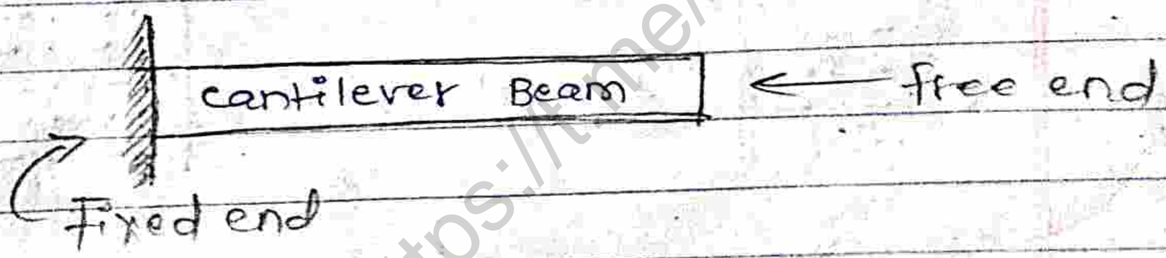


# TYPES OF BEAM

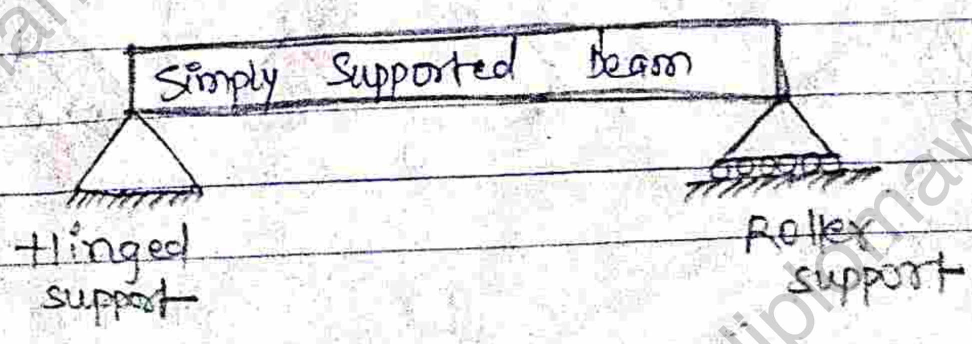
Beams are classified on the basis of Nature of support and the overhanging lengths from support.

- Cantilever Beam
- simply supported Beam
- Over Hanging Beam
- fixed Beam
- Continous Beam
- proposed cantilever Beam

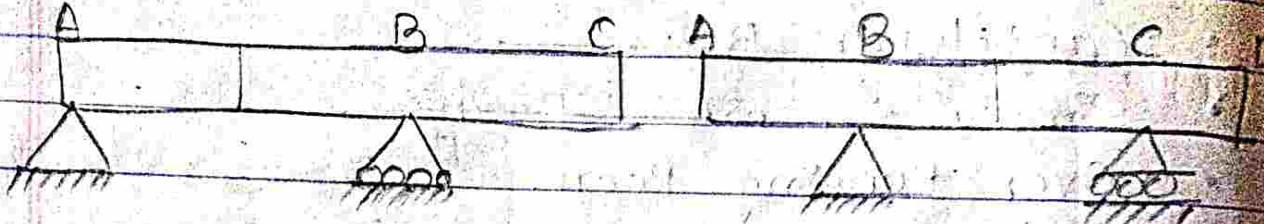
i) Cantilever Beam — A beam having its one end fixed or built in and the other end free is called a cantilever beam.




Simply supported Beam — A beam supported by a hing at one end and a roller at the other end is called a simply supported Beam

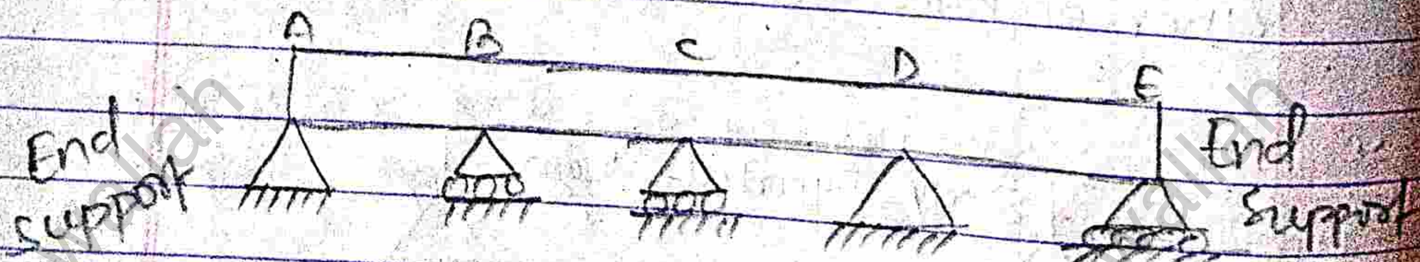


iii) Over hanging Beam :- if one or both ends of a beam projects beyond or extends over the supports then it is called over hanging beam.

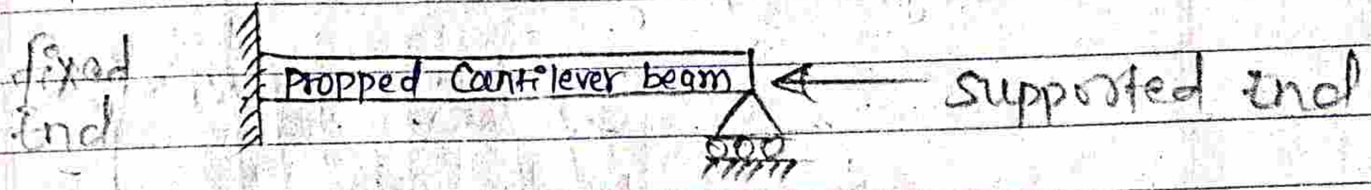



iv) fixed Beam :- A beam whose both ends are rigidly fixed or built in to the walls of column is known as fixed beam.


v) Continous Beam :- A beam which has more than two supports is called Continous Beam.



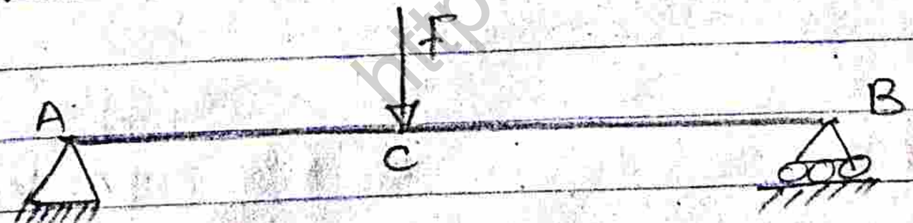
vi) Propped Cantilever Beam :— When a support is provided at some point of a cantilever beam in order to resist deflection of the beam, is called propped cantilever beam.




## CLASSIFICATION OF LOADS

(i) concentrated loads or point loads
(ii) Uniformly distributed loads
(iii) uniformly varying loads.

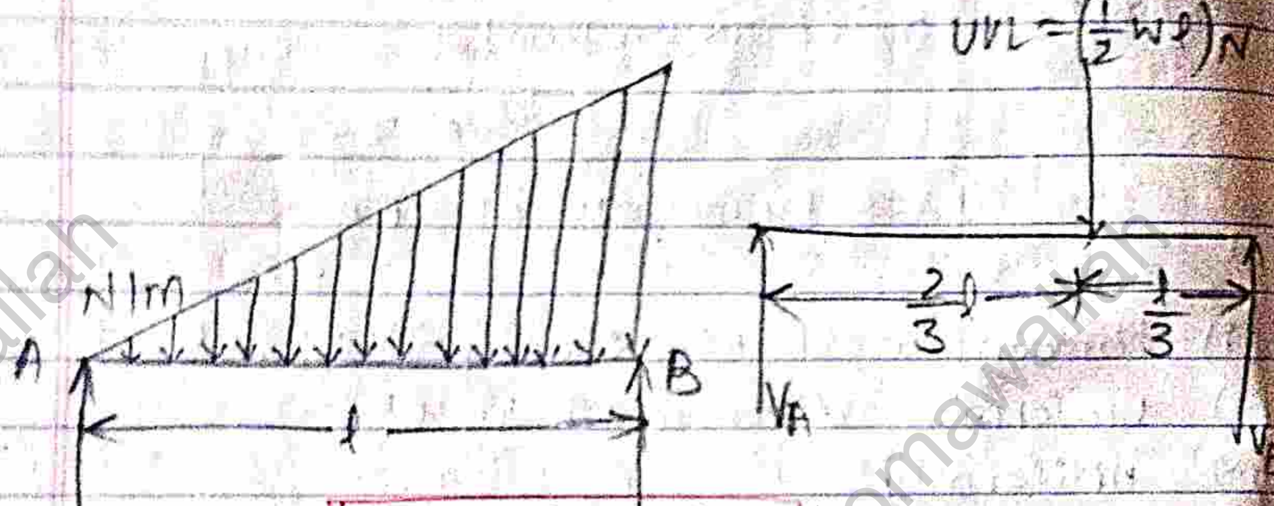
i) Concentrated load or point load —
if the applied loads is acting at a point of beam or rigid body, then the load is known as concentrated load.




(ii) Uniformly distributed load (UDL) :—
if the applied loads of uniform intensity (force per unit of length of beam) is spread over a length of the beam. it is known as Uniformly distributed load (UDL).

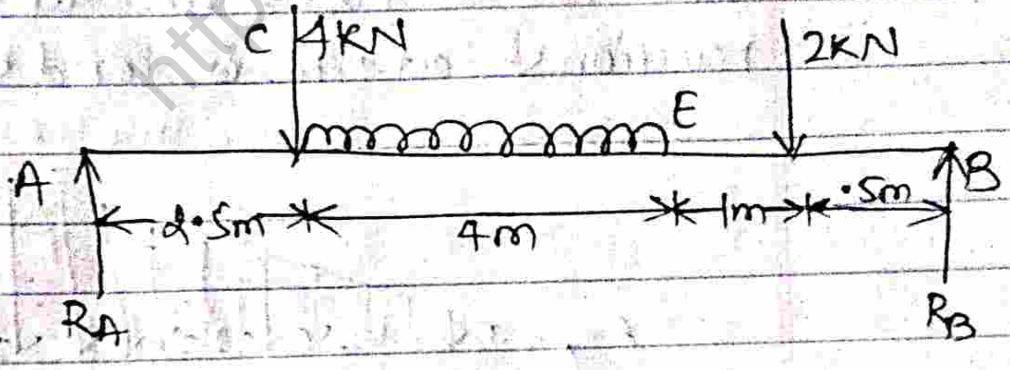

(iii) Uniformly varying load (UVL) :-

If the intensity of loading varies linearly from one location of beam to another location of the beam, it is known as Uniformly varying load.

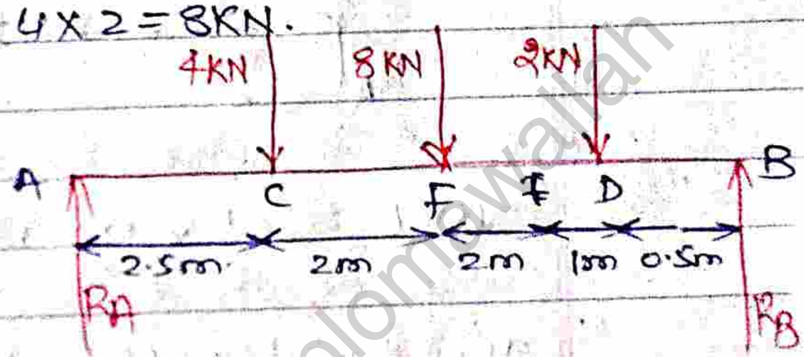



$UVL = \left(\frac{1}{2} w l\right) N$

Q. A simply supported beam AB, of span 8m is loaded. Determine reaction $R_A$ and $R_B$ of the beam.




Total load of UDL = 4 × 2 = 8KN.




$\Sigma f_y = 0$

$4 + 8 + 2 = R_A + R_B$

$R_A + R_B = 14.$

$\Sigma M_A = 0$

$-(4 \times 2.5) - (8 \times 4.5) - (2 \times 7.5) + (R_B \times 8) = 0$

$8R_B = 61$

$R_B = \frac{61}{8} = 7.63KN.$

$\boxed{R_B = 7.63KN}$.

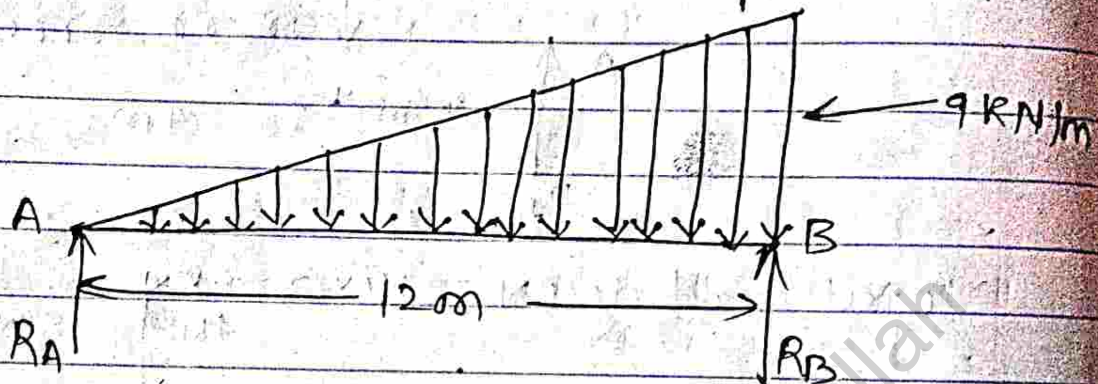
$R_A + R_B = 14$

$R_A + 7.63 = 14$

$R_A = 14 - 7.63$

$R_A = 6.37 \ KN.$

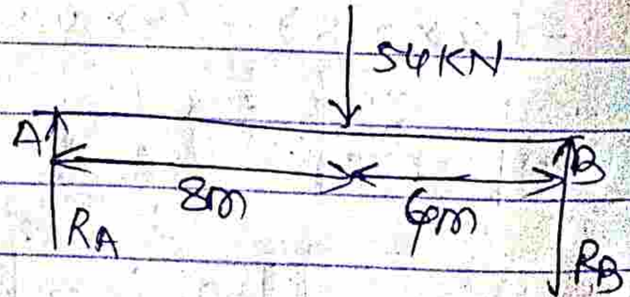
Q. A simply supported beam of span 12m carries a uniformly varying load from zero at end A to 9KN/m at end B. calculate reactions at two ends of the support.




$$UVL = \frac{1}{2} \times 9 \times 12 = 54KN.$$

This load will act from cog of the load triangle at a distance of $\frac{2}{3} \times l$.

Distance $= \frac{2}{3} \times 12 = 8m$ from end A.




$\Sigma f_y = 0$

$R_A + R_B = 54$

$\Sigma M_A = 0$

$R_A \times 0 - (54 \times 8) + R_B \times 12 = 0$

$\cancel{12 R_B = 54 \times 8}$

$R_B = \frac{\cancel{54}^{18} \times \cancel{8}^{2}}{\cancel{12}_{3}} = 36KN$

$R_A + R_B = 54 \Rightarrow \boxed{R_A = 18KN}$

UNIT - 3 Friction

## Friction

It is the phenomenon, in which the motion between two contacting bodies is opposed due to irregularities on the contacting surfaces, is known as friction.

Why?
To oppose the relative motion of the body.

When?
When bodies/body and surface are in contact.

I) Body will try to come in relative motion after applying force (Rest)

II) Body will come in relative motion after applying force (motion).

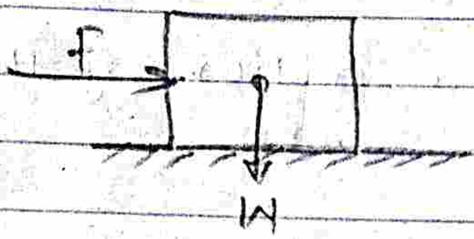
Where?
Acting tangentially in contact surface/contact point.

Friction

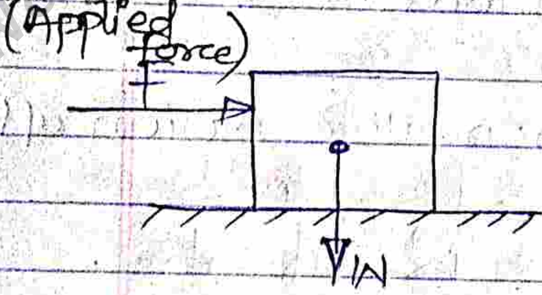
- Static friction
  - Body in Rest
  - Static friction force depends on Applied force.
  - Rest up to limiting condition
- Kinectic friction
  - Body in motion
  - Kinectic Friction does not depend on Applied force.
  - motion after limiting friction.

Before limiting condition (Rest) | At limiting condition | After limiting condition

Body just start
Begin to motion

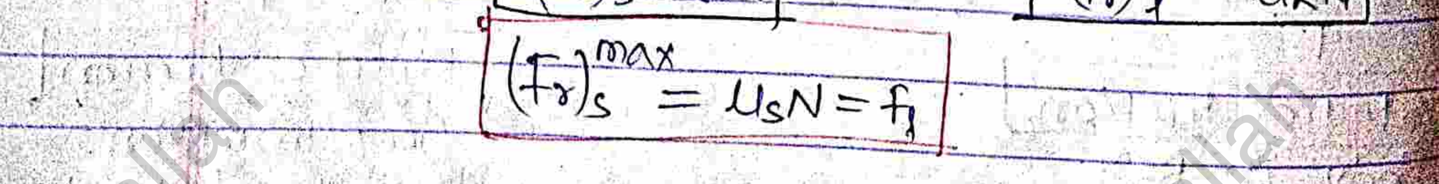



Limiting condition – At which the body will just start to move.




$F \rightarrow (F_1, F_2, F_l)$

① Before limiting condition ② At limiting condition ③ After limiting condition




$(F_r)_{s_1} = f_1$

$\boxed{(F_r)_s^{max} \propto N}$

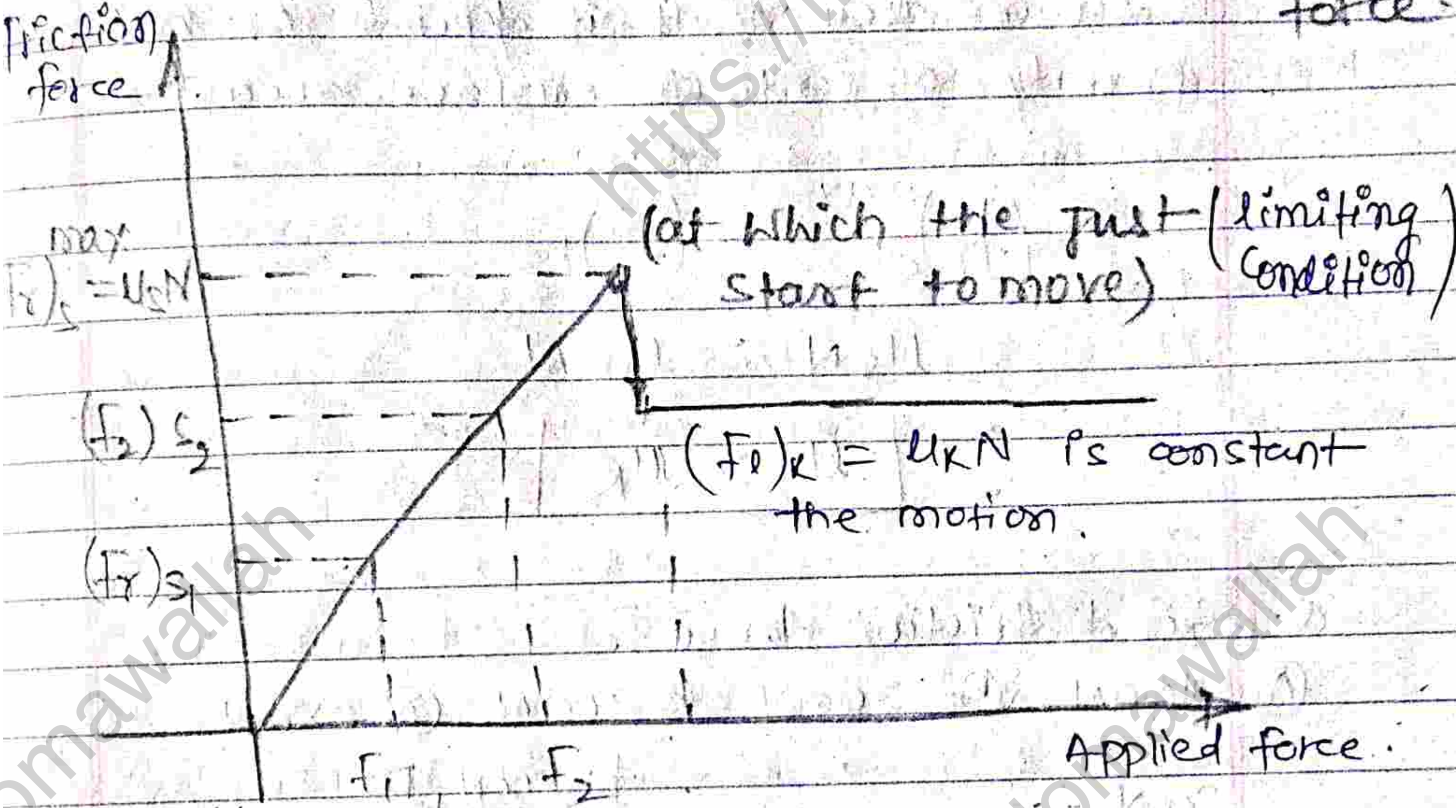
$\boxed{(F_r)_s^{max} = \mu_s N = f_l}$

$\boxed{(F_r)_l = \mu_k N}$

$\mu_s$ = static coefficient of friction

$\mu_k$ = Kinetic coefficient of friction.

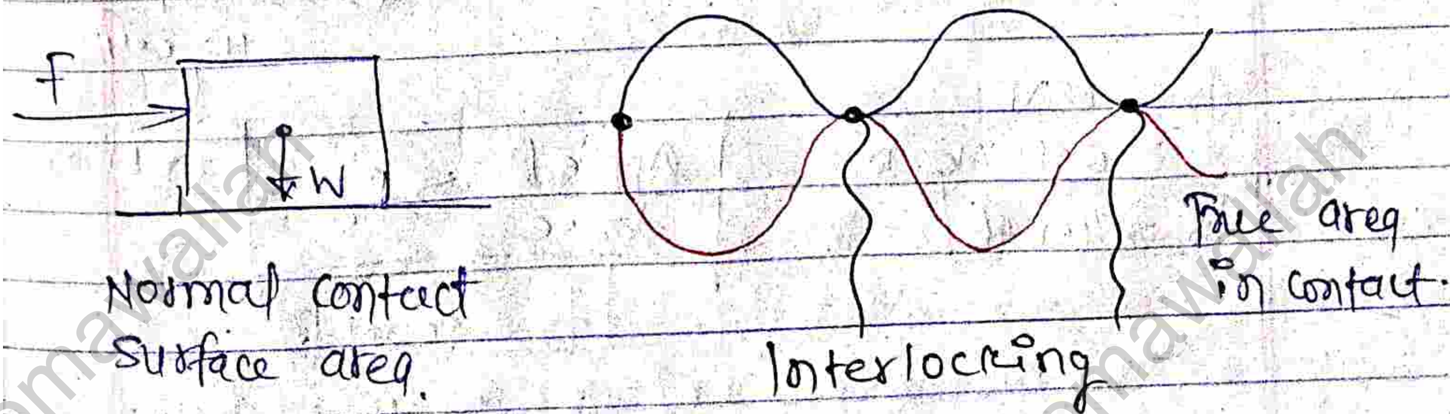
## Graph for applied force v/s friction force.




ssumption — Kinectic friction is independent of velocity

Note :- ① At low velocity, Actually kinectic friction is constant
② At high velocity, kinectic friction is decreased.

$$\boxed{\text{Static friction force} > \text{Kinectic friction force}}$$



Note – Friction does not depend on contact surface area. But it depends on nature property of contact surface area.

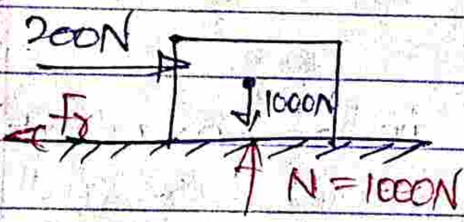
$$(F_r)_s^{max} > (F_r)_k$$

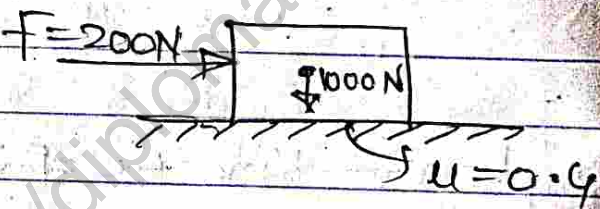
$$\mu_s N > \mu_k N$$

$$\boxed{\mu_s > \mu_k}$$

Q. find friction force?

(A) 400N (B) 200N (C) 100N (D) 250N



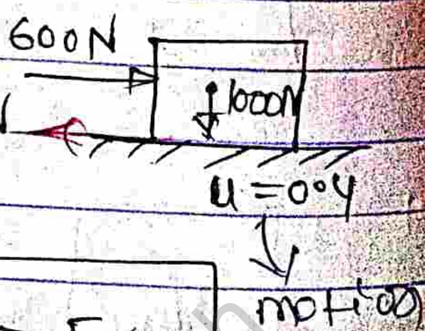

$F_r = \mu N$
$= 0.4 \times 1000 = 400N$.

Applied force $\cancel{>}$ $F_r$

$$\boxed{F_r = \text{Applied} = 200N}$$

Q. find friction force –

(A) 400N (B) 600N (C) 100N (D) 250N




$F_r = \mu N$
$= 0.4 \times 1000$

$$\boxed{F_r = 400N}$$

Applied force $> F_r$

$$\boxed{F_r = 400N}$$

$\boxed{F_r = \mu N}$ — Limiting friction force / Max static friction force.

$\mu_s \rightarrow$ static c.o.f

$\mu_k \rightarrow$ Kinectic c.o.f

** $\boxed{F_{applied} = \mu N_s = (F_r)_s^{max}}$ (Rest)

* $\boxed{F_{applied} < \mu_s N}$ (Rest)

* $\boxed{F_{applied} > \mu_s N}$ (motion)

$(F_r)_s^{max} = \mu_s N \Rightarrow$ limiting friction force.

$\mu \rightarrow \mu_s$ (By default)

$F_r = \mu N \rightarrow (F_r)_s^{max} = \mu_s N$ (By default)

~~(1) Friction angle ($\phi$) & Resultant Reaction (R).~~

TYPE OF FRICTION

- Dry (coulomb's) friction
  - Static friction
  - Kinectic friction
- fluid friction

1. Dry (coulomb's) friction — The frictional force that resists relative lateral motion of ~~two~~ unlubricated solid surfaces in contact is called coulomb's dry friction.

1. Static friction – It is the Friction between two stationary surfaces which have a tendency to relative motion due to external forces.

2. Kinectic friction (Dynamic friction) – It is the friction between two moving Surface in contact.

2. fluid friction – fluid friction, also called lubricated friction, resists relative lateral motion of two solid surfaces seperated by a layer of gas or liquid.

## Law of Friction

- law of static Friction –

I The frictional force always acts in a direction opposite to that in which the body tends to move.

II The frictional force is directly proportional to the normal reaction between the two contact surfaces.

III The friction force depends upon the nature of the surfaces in contact.

IV It is independent of the area and shape of the contacting surfaces.

• Laws of Dynamic or Kinetic friction

I. The Frictional force always acts in a direction opposite to that in which the body moves.

II. The Frictional force is directly proportional to the normal reaction between the two contact surfaces.

III. The frictional force remains constant for moderate speeds but it decreases slighty with the increase of speed.

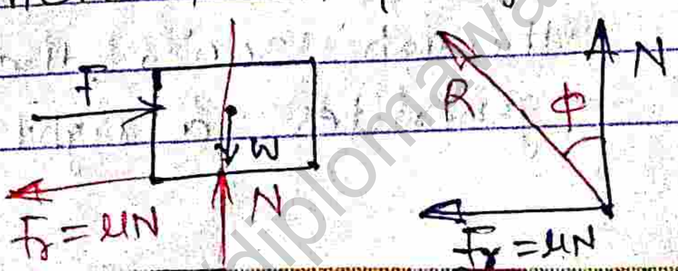
• Coefficient of friction – The ratio of force of friction to the normal reaction between the contact surfaces.

$$\boxed{\mu = \frac{F_r}{N}}$$

• Angle of Friction – The angle which the resultant of normal reaction and limiting force of friction makes with the normal reaction.

$$\boxed{\tan\phi = \frac{F_r}{N}}$$

Where $\phi$ = Angle of friction



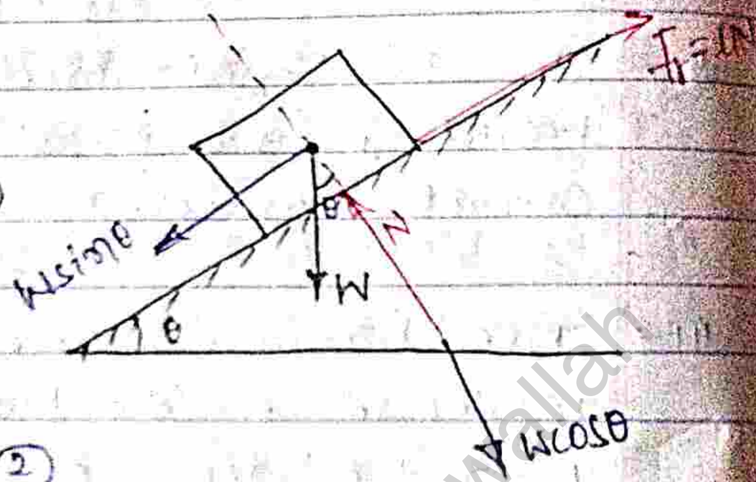

$$\boxed{\mu = \tan\phi}$$

- Angle of Repose (θ) — The angle made by inclined plane with horizontal at which the body/block just start sliding down on the inclined plane.

$\Sigma f_x = 0$

$\mu N - W\sin\theta = 0$

$W\sin\theta = \mu N$ ---- ①




$\Sigma f_y = 0$

$N - W\cos\theta = 0$

$W\cos\theta = N$ ---- ②

On solving Eqn ① and Eqn ②, we get.

$$\frac{W\sin\theta}{W\cos\theta} = \frac{\mu N}{N}$$

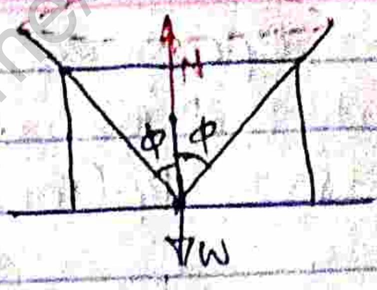
$$\tan\theta = \mu$$

$$\boxed{\theta = \tan^{-1}(\mu)}$$ At limiting condition

→ $\boxed{\phi = \theta}$ At limiting condition.

- Friction cone — When the direction of applied force 'F' is gradually changed through 360°, then the resultant 'R' generates a right circular cone.

$a \rightarrow$ projected angle of cone.




$$\boxed{a = 2\phi}$$

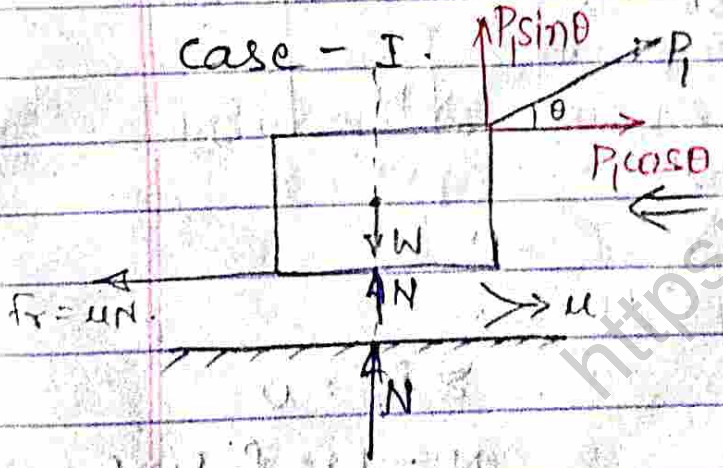
- Radius of friction cone $\rightarrow f_r = \mu N$.

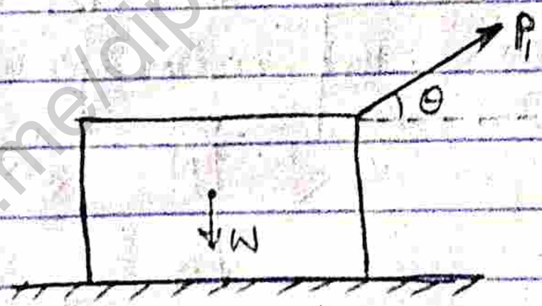
## # Concept of pulling & pushing force –

* $P_1 / P_2 \rightarrow \min = ?$ ; $P_1 \rightarrow$ pulling force will be minimum.

* At which angle θ the pulling force is least.

Case – I.




$\Sigma f_x = 0$;

$P_1 \cos\theta - \mu N = 0$

$P_1 \cos\theta = \mu N$ ---- (i)

$\Sigma f_y = 0$

$P_1 \sin\theta + N - W = 0$

$N = W - P_1 \sin\theta$

from eqⁿ (i), putting value of N, we get

$$\frac{P_1 \cos\theta}{\mu} = W - P_1 \sin\theta$$

$$P_1 \cos\theta = \mu (W - P_1 \sin\theta)$$

$$P_1 \cos\theta = \mu W - \mu P_1 \sin\theta$$

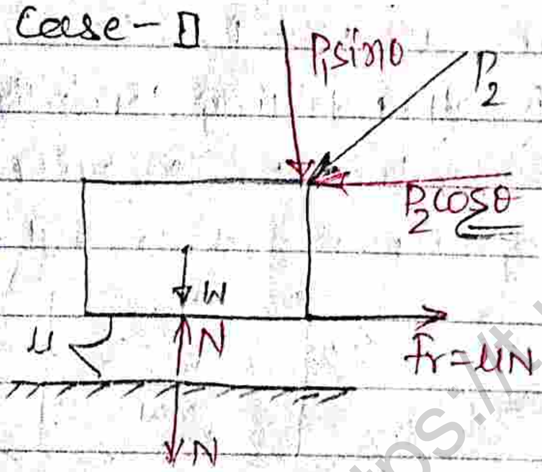
$$P_1 \cos\theta + \mu P_1 \sin\theta = \mu W$$

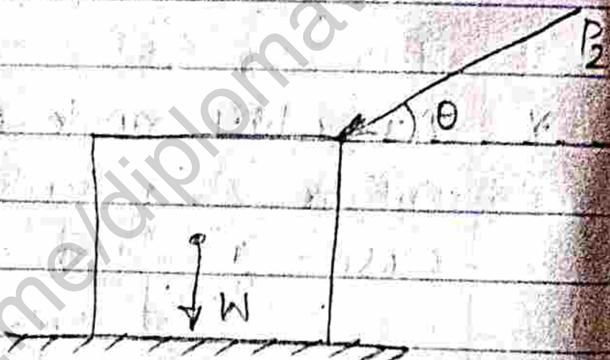
$$P_1 (\cos\theta + \mu \sin\theta) = \mu W$$

$$\boxed{P_1 = \frac{\mu W}{\cos\theta + \mu \sin\theta}}$$

$$P_1 \propto \frac{1}{\cos\theta + \mu \sin\theta}$$

Case – II




$$\Sigma f_x = 0$$

$$\mu N - P_2 \cos\theta = 0$$

$$\mu N = P_2 \cos\theta \quad \text{— (ii)}$$

$$\Sigma f_y = 0$$

$$N - W - P_2 \sin\theta = 0$$

$$N = W + P_2 \sin\theta$$

from eqⁿ (ii); putting the value of N.

$$P_2 \cos\theta = \mu (W + P_2 \sin\theta)$$

$$P_2 \cos\theta = \mu W + \mu P_2 \sin\theta$$

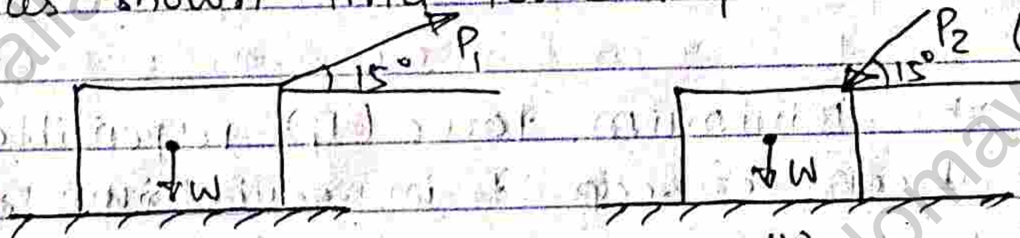
$$\mu W = P_2 \cos\theta - \mu P_2 \sin\theta$$

$$\mu W = P_2(\cos\theta - \mu\sin\theta)$$

$$\boxed{P_2 = \frac{\mu W}{\cos\theta - \mu\sin\theta}}$$

$$P_2 \propto \frac{1}{\cos\theta - \mu\sin\theta}$$

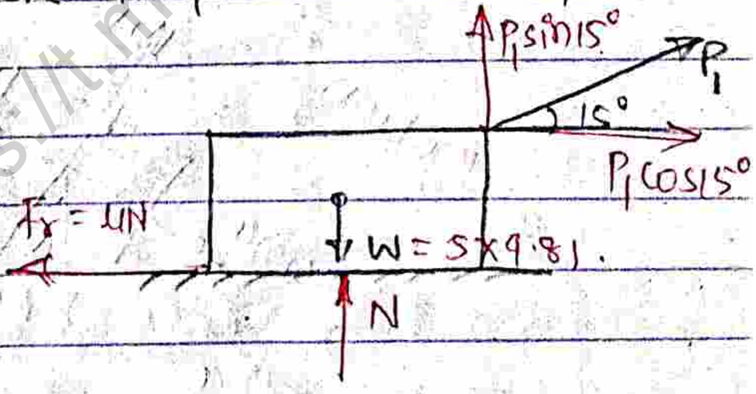
Q. A wooden block rests on a horizontal plane as shown. Find force required to (a) pull it? (b) push it?




(a) (b)

Assume mass of the block is 5kg and coefficient of friction b/w block & plane is 0.4.

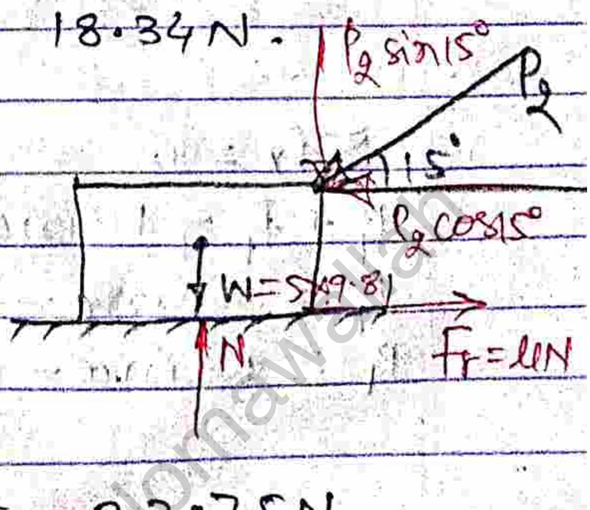
Sol$^n$:- pulling $(P_1)$ —




$$P_1 = \frac{\mu W}{\cos\theta + \mu\sin\theta}$$

$$= \frac{0.4 \times 5 \times 9.81}{\cos 15^\circ + 0.4\sin 15^\circ} = 18.34\,N.$$

• pushing $(P_2)$ —




$$P_2 = \frac{\mu W}{\cos\theta - \mu\sin\theta}$$

$$= \frac{0.4 \times 5 \times 9.81}{\cos 15^\circ - 0.4\sin 15^\circ} = 22.75\,N.$$

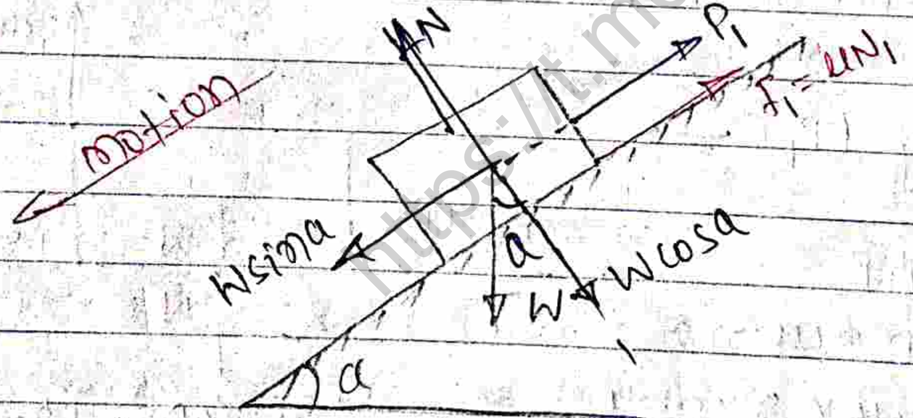
## Equilibrium of Body on Inclined plane

If the Inclination of the plane to the horizontal is less than the angle of repose, the body will be in equilibrium without the aid of any external force for the purpose.

(1) The external force P is parallel to the plane.
(2) The external force P is horizontal.

# Force parallel to Inclined plane.

Case - I → Minimum force ($P_1$) responsible for the block to keep it in equilibrium at the point of impeding downwards motion along the inclined plane.




$\Sigma f_x = 0$

$P_1 + f_1 - W\sin\alpha = 0$

$P_1 = W\sin\alpha - f_1$

$P_1 = W\sin\alpha - \mu N_1$ --- (1)

$\Sigma f_y = 0$

$N_1 = W\cos\alpha$

putting the value of $N_1$ in eqn (i) :

$$P_1 = W\sin a - \mu N_1$$
$$P_1 = W\sin a - \mu(W\cos a)$$
$$P_1 = W\sin a - \mu W\cos a$$
$$P_1 = W(\sin a - \mu\cos a)$$

Subsitiuting the value of $\mu = \tan\phi$ in above eqn.

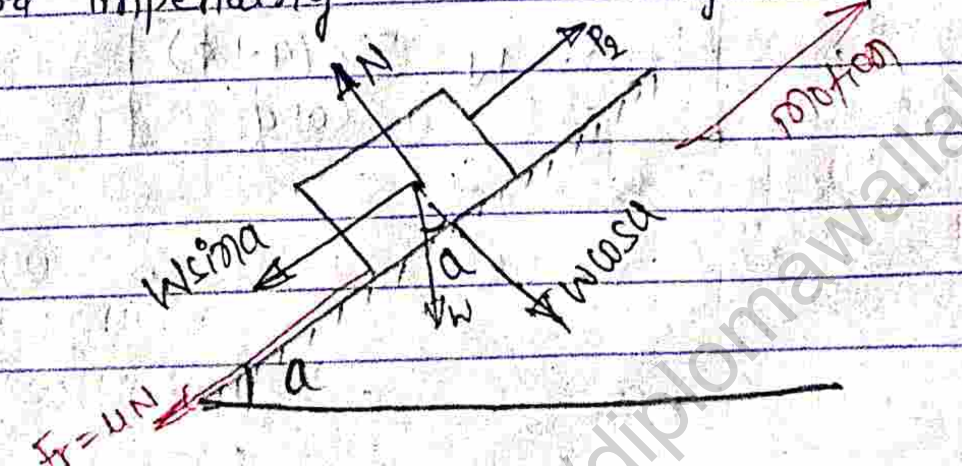
$$P_1 = W(\sin a - \tan\phi\cos a) \quad [\because \mu = \tan\phi]$$

On multiply both side by $\cos\phi$; we get

$$P_1\cos\phi = W(\sin a\cos\phi - \sin\phi\cos a)$$
$$P_1\cos\phi = W\sin(a-\phi)$$

$$\boxed{P_1 = \frac{W\sin(a-\phi)}{\cos\phi}}$$

Case-II → Maximum force $P_2$ which will keep the block in equilibrium at the point of upward impending motion along the plane.



$\Sigma f_x = 0$ $\quad\quad$ $\Sigma f_y = 0$

$W\sin a + F_r = P_2$ $\quad\quad$ $N = W\cos a$

$P_2 = W\sin a + F_r$

$P_2 = W\sin a + \mu N$ —— (i)

putting the value of N in eq$^n$ (i) ; we get.

$P_2 = W\sin a + \mu N$

$P_2 = W\sin a + \mu(W\cos a)$

$P_2 = W\sin a + \mu W\cos a$

substituent the value of $\mu = \tan\phi$ in above.

$P_2 = W\sin a + \mu W\cos a$

$P_2 = W\sin a + \tan\phi\, W\cos a$

$P_2 = W(\sin a + \tan\phi \cos a)$

On multiply ~~on~~ the both side by $\cos\phi$.

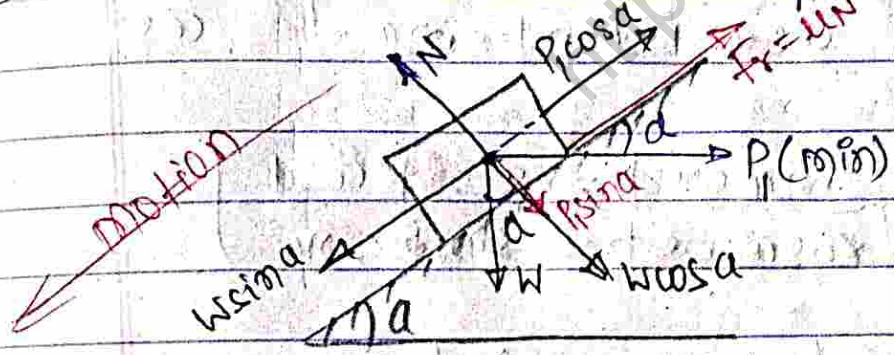
$\cos\phi\, P_2 = W(\sin a\cos\phi + \sin\phi\cos a)$

$\cos\phi\, P_2 = W\sin(a+\phi)$

$$\boxed{P_2 = W\,\frac{\sin(a+\phi)}{\cos\phi}}$$

# Force Horizontal :

Case-I : Minimum force ($P_1$).




$\Sigma f_x = 0$

$P_1 \cos a + F_r - W \sin a = 0$

$P_1 \cos a = W \sin a - F_r$

$P_1 \cos a = W \sin a - \mu N$ $\quad [\because F_r = \mu N]$

$\quad --- (i)$

$\Sigma f_y = 0$

$N - P_1 \sin a - W \cos a = 0$

$N = W \cos a + P_1 \sin a$

putting the value of N in eq$^n$ (i), we get

$P_1 \cos a = W \sin a - \mu N$

$P_1 \cos a = W \sin a - \mu (W \cos a + P_1 \sin a)$

$P_1 \cos a = W \sin a - \mu W \cos a - \mu P_1 \sin a$

$P_1 \cos a + \mu P_1 \sin a = W \sin a - \mu W \cos a$

$P_1 (\cos a + \mu \sin a) = W (\sin a - \mu \cos a)$

$$\boxed{P_1 = W \left( \frac{\sin a - \mu \cos a}{\cos a + \mu \sin a} \right)}$$

Subsituent the value of $u = \tan\phi$ in above Eq

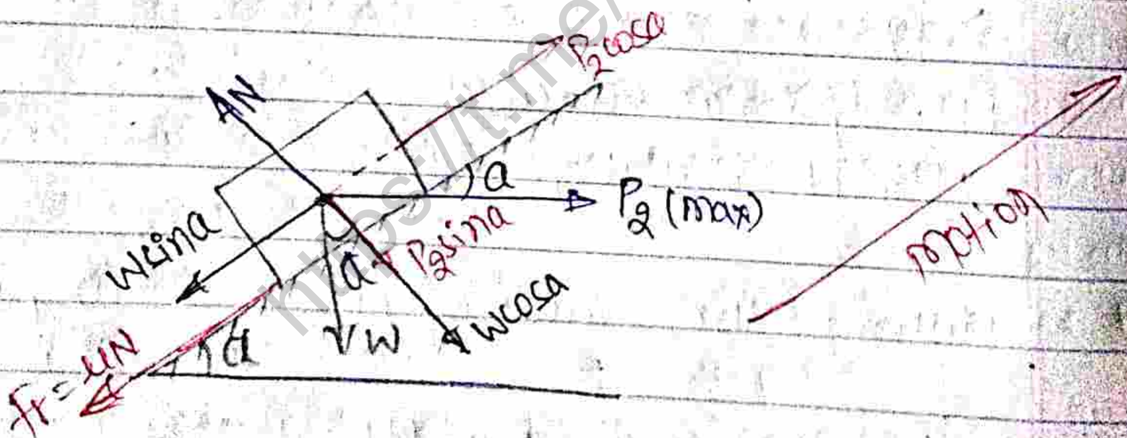
$$P_1 = W\left(\frac{\sin a - \tan\phi\cos a}{\cos a + \tan\phi \sin a}\right) \times \frac{\cos\phi}{\cos\phi}$$

$$P_1 = W\left(\frac{\sin a\cos\phi - \sin\phi\cos a}{\cos a\cos\phi + \sin\phi \sin a}\right)$$

$$P_1 = W\left[\frac{\sin(a-\phi)}{\cos(a-\phi)}\right]$$

$$\boxed{P_1 = W\tan(a-\phi)}$$

Case – II Maximum force ($P_2$) –




$\Sigma f_x = 0$

$W\sin a + fr = P_2\cos a$

$P_2\cos a = W\sin a + fr$

$P_2\cos a = W\sin a + uN$ --- (i) [$\because fr = uN$]

$\Sigma f_y = 0$

$N - P_2 \sin a - W\cos a = 0$

$N = P_2 \sin a + W\cos a$.

putting the value of N in eq^n (i).

$P_2 \cos a = W\sin a + \mu N$

$P_2 \cos a = W\sin a + \mu (P_2 \sin a + W\cos a)$

$P_2 \cos a = W\sin a + \mu P_2 \sin a + \mu W\cos a$

$P_2 \cos a - \mu P_2 \sin a = W\sin a + \mu W \cos a$

$P_2 (\cos a - \mu \sin a) = W(\sin a + \mu \cos a)$

$$\boxed{P_2 = W\left(\frac{\sin a + \mu \cos a}{\cos a - \mu \sin a}\right)}$$

Subsituent the value of $\mu = \tan\phi$ in above eq^n.

$$P_2 = W\left(\frac{\sin a + \tan\phi \cos a}{\cos a - \tan\phi \sin a}\right) \times \frac{\cos\phi}{\cos\phi}$$

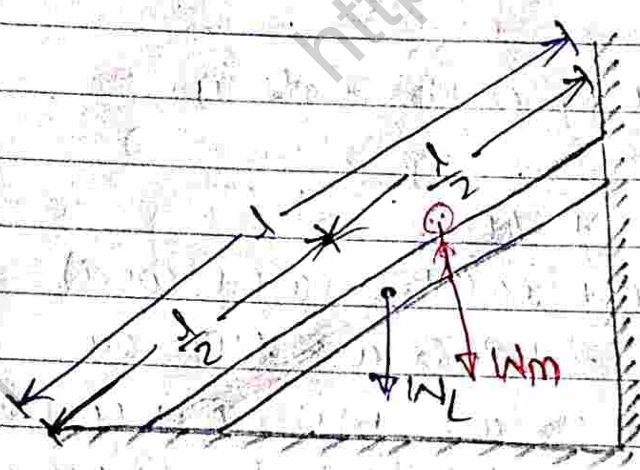
$$P_2 = W\left(\frac{\sin a \cos\phi + \sin\phi \cos a}{\cos a \cos\phi - \sin\phi \sin a}\right)$$

$$P_2 = W\frac{\sin(a+\phi)}{\cos(a+\phi)}$$

$$\boxed{P_2 = W\tan(a+\phi)}$$

# LADDER FRICTION

The ladder is used to climb or to scale the roofs or walls.

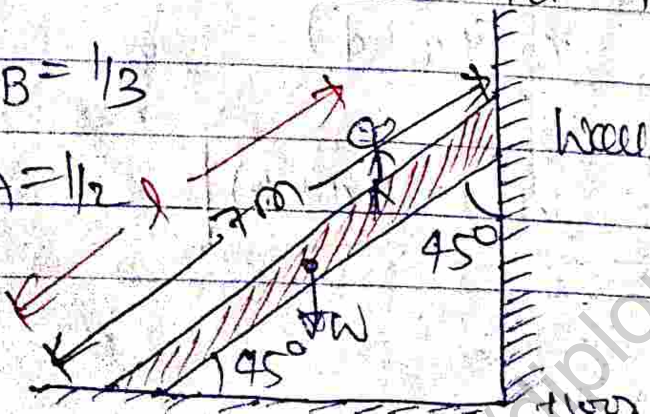


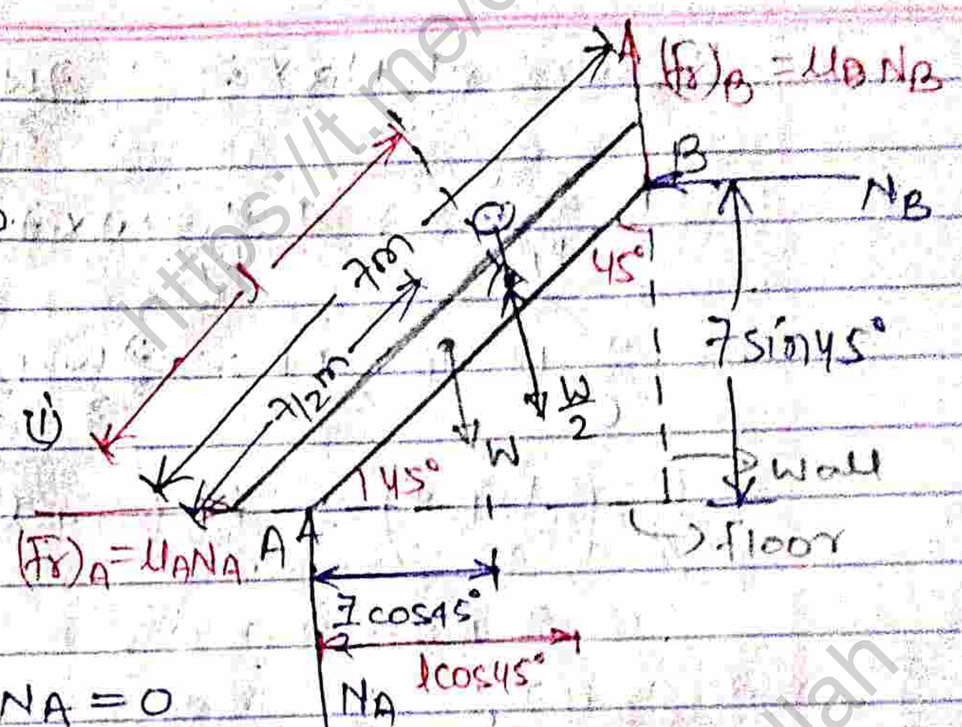

$W_l$ = weight of the ladder
$W_m$ = weight of the men

Q. A 7m long ladder rests against a wall with which it makes an angle of 45° and on a floor. if a man whose weight is one half of that of the ladder. climbs it, at what distance along the ladder will be . when the ladder is about to slip?
the coefficient of friction (c.o.f) between ladder and the wall is $\frac{1}{3}$ and that between the ladder and the floor is $\frac{1}{2}$.

$W_m = \frac{W}{2}$
$W_R = W$

$\mu_B = 1/3$
$\mu_A = 1/2$






$\Sigma f_x = 0$

$-N_B + \mu_A N_A = 0$

$-N_B + \frac{N_A}{2} = 0$

$\boxed{N_A = 2N_B}$ --- (i)

$\Sigma f_y = 0$

$\mu_B N_B - W - \frac{W}{2} + N_A = 0$

$\frac{N_B}{3} - \frac{3W}{2} + N_A = 0$

$\frac{N_B}{3} + 2N_B = \frac{3W}{2}$

$\frac{7N_B}{3} = \frac{3W}{2}$

$\boxed{N_B = \frac{9}{14}W}$

Now,

$N_A = 2N_B$

$= 2 \times \frac{9}{14} W$

$\boxed{N_A = \frac{9}{7}W}$

$\Sigma M_A = 0$

$(Fr)_A \times 0 + (N_A \times 0) - W \times \frac{7}{2}\cos 45° - \frac{W}{2} \times l\cos 45° + (Fr)_B \times 7\cos 45°$

$+ N_B \times 7\sin 45° = 0$

$-W \times \frac{7}{2} \times \frac{1}{\sqrt{2}} - \frac{W}{2} \times l \times \frac{1}{\sqrt{2}} + (Fr)_B \times \frac{7}{\sqrt{2}} + N_B \times \frac{7}{\sqrt{2}} = 0$

$(Fr)_B \times 7 \times \frac{1}{\sqrt{2}} + N_B \times 7 \times \frac{1}{\sqrt{2}} = \frac{W \times 7}{2\sqrt{2}} + \frac{Wl}{2\sqrt{2}}$

$\cancel{\frac{1}{\sqrt{2}}}\left[(Fr)_B \times 7 + N_B \times 7\right] = \cancel{\frac{1}{\sqrt{2}}}\left(\frac{7W}{2} + \frac{Wl}{2}\right)$

$$\mu_B N_B \times 7 + N_B \times 7 = \frac{7W}{2} + \frac{W}{2}l$$

$$\frac{1}{3} \times \frac{9}{14} W \times 7 + \frac{9}{14} \times W \times 7 = \frac{7W}{2} + \frac{Wl}{2}$$

$$\frac{9}{6} W + \frac{9}{2} W = \frac{7W}{2} + \frac{Wl}{2}$$

$$\cancel{\frac{W}{2}}\left(\frac{9}{3} + 9\right) = \cancel{\frac{W}{2}}(7 + l)$$

$$\frac{9}{3} + 9 - 7 = l$$

$$l = \frac{9}{3} + 2 = 3 + 2$$

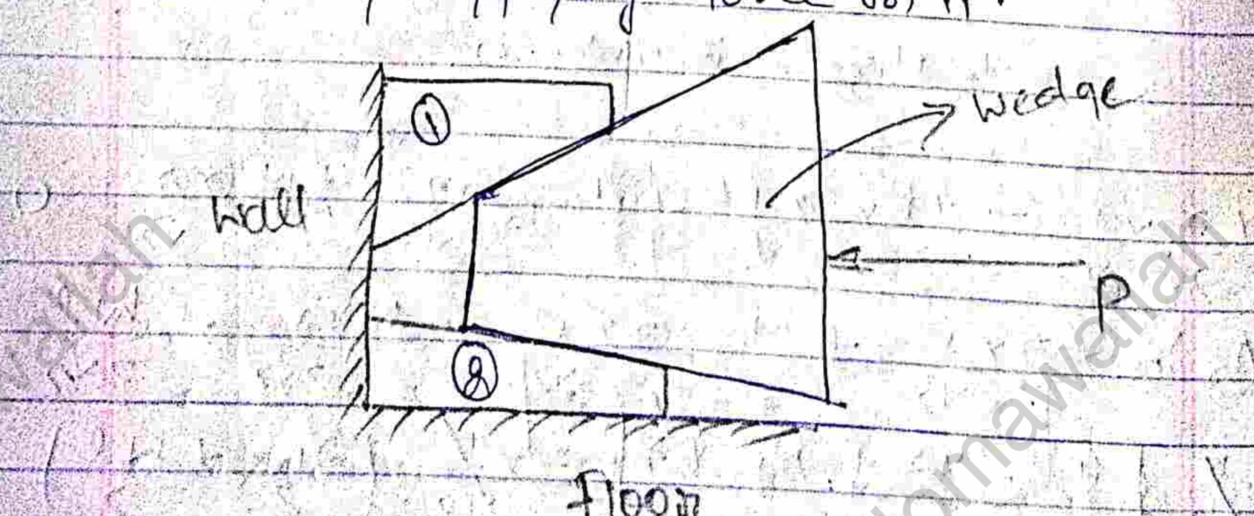
$$\boxed{l = 5m}$$

## WEDGE

Wedge is an object which is inserted between two bodies to either hold them in a place.

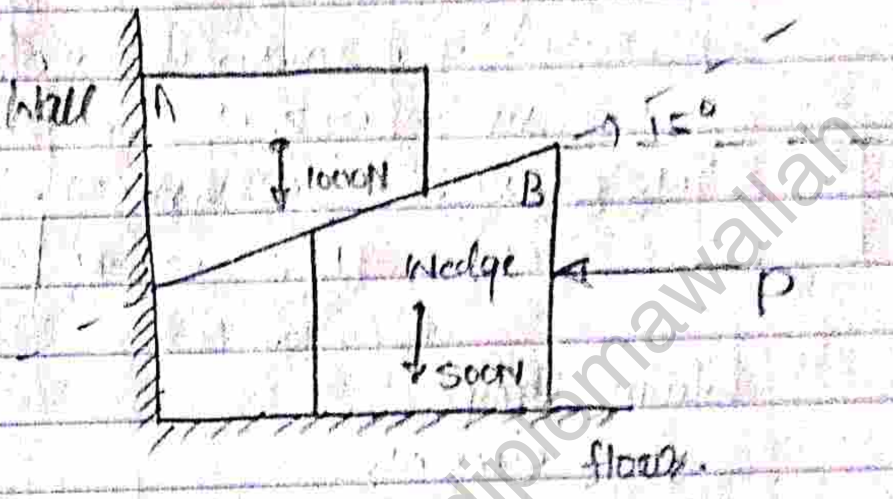
Or

It is also used to move one relative to other by applying force on it.

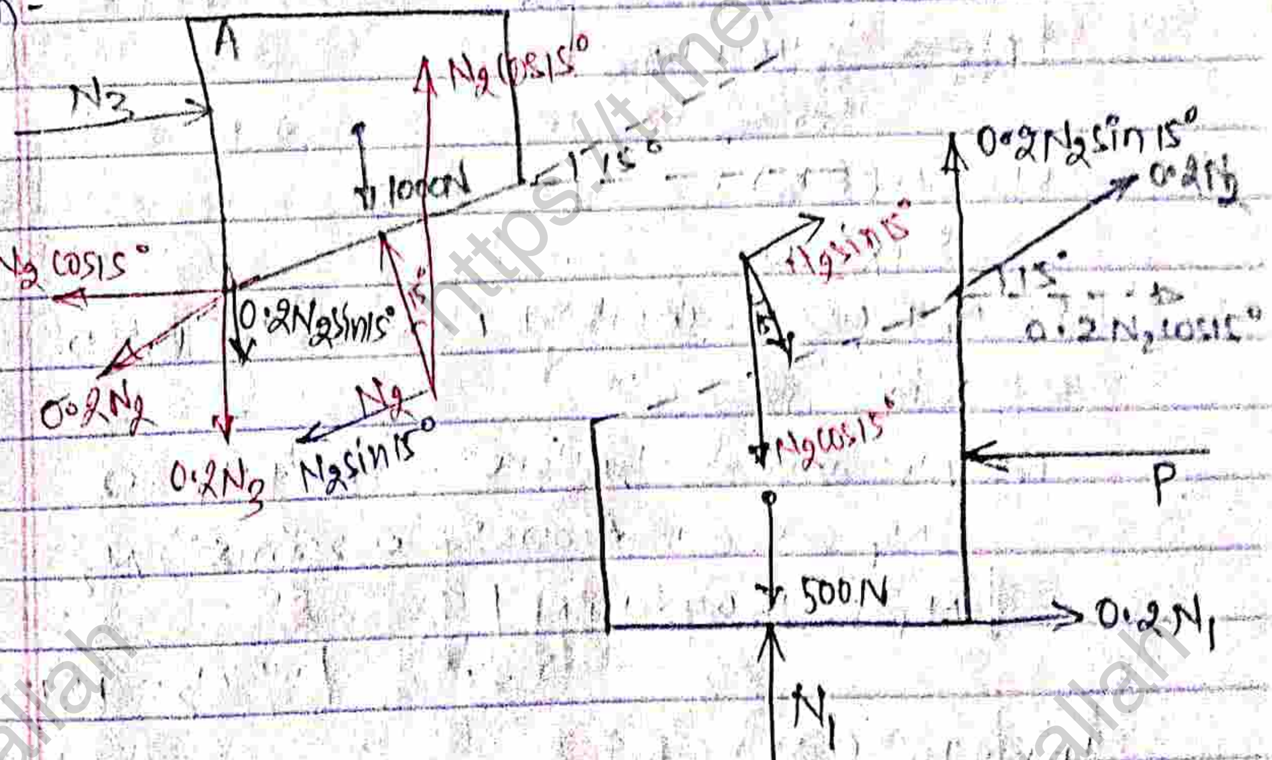


Q. A block weighing 1000N is to be raised by means of 15° wedge B weighing 500N as shown in fig. Assuming the c.o.f between contact surfaces to be 0.2. Determine what minimum horizontal force 'P' should be applied to raise the block?

$\mu = 0.2$




solution-



Block (A).

$\Sigma f_x = 0$

$N_3 - N_2 \sin 15^\circ - 0.2 N_2 \cos 15^\circ = 0$

$N_3 - N_2 (\sin 15^\circ + 0.2 \cos 15^\circ) = 0$

$N_3 = 0.45 N_2$ —— ①

$\Sigma f_y = 0$

$N_2 \cos 15^\circ - 0.2 N_2 \sin 15^\circ - 0.2 N_3 - 1000 = 0$

$N_2 \cos 15^\circ - 0.2 N_2 \sin 15^\circ - 0.2 \times 0.45 N_2 = 1000$

$N_2 (\cos 15^\circ - 0.2 \sin 15^\circ) - 0.2 \times 0.45 = 1000$

$\boxed{N_2 = 1213.35 N}$

from Eqn ①

$N_3 = 0.45 N_2$

$= 0.45 \times 1213.35$

$\boxed{N_3 = 546.01 N.}$

WEDGE (B).

$\Sigma f_x = 0$

$0.2 N_1 + 0.2 N_2 \cos 15^\circ + N_2 \sin 15^\circ - P = 0$

$\Sigma f_y = 0$

$0.2 N_2 \sin 15^\circ - N_2 \cos 15^\circ - 500 + N_1 = 0$

$N_1 = 500 + (\cos 15^\circ - 0.2 \sin 15^\circ) N_2$

$\boxed{N_1 = 1609.19 N}$. $[\because N_2 = 1213.35 N]$

from Eqn ②

$0.2 N_1 + 0.2 N_2 \cos 15^\circ + N_2 \sin 15^\circ - P = 0$

$0.2 \times 1609.19 + 0.2 \times 1213.35 \cos 15^\circ + N_2 \sin 15^\circ = P$

$\boxed{P = 870.29 N}$ Min. Horizontal force.

# UNIT-4 COG, COM & CENTROIDS

Rigid Body — The composed of number of very small particles occupying the fixed position.

**Rigid Body**

- **Mass (m)** — Quantity of matter or material.
- **Weight (W)** — • Gravitational force applied by the earth on the body.
- **Geometry of body** — Shape size of body
  - Volume (V)
  - Area (A)
  - length (L).

* **Centre of mass — (COM) —**
  - It related to mass of the body.
  - It is the point where entire mass of the body is assumed to be concentrated.

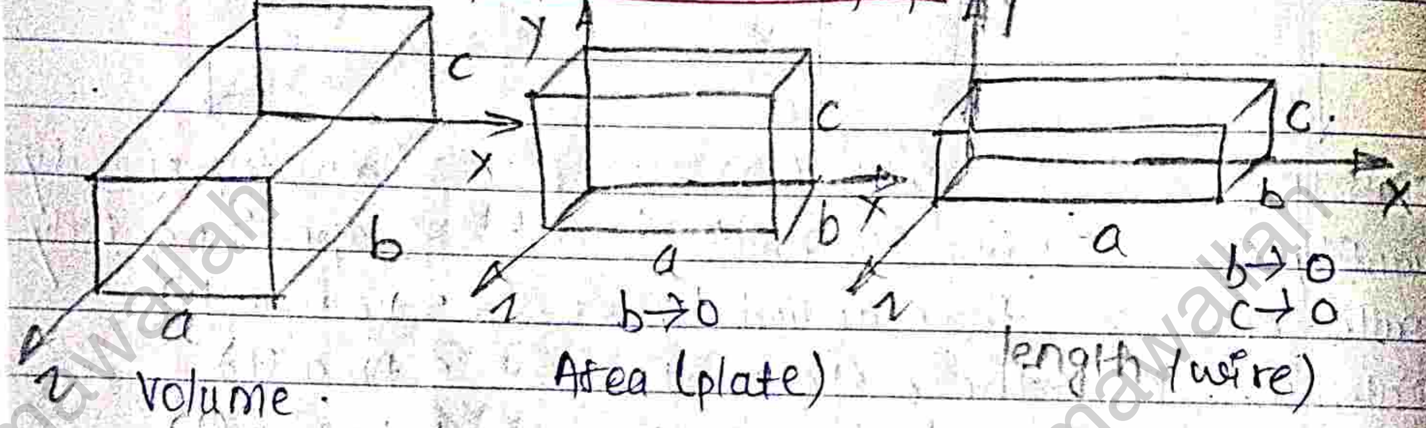
* **Centre of gravity (COG) —**
  - It related to weight of the body.
  - It is the point where entire weight of body is assumed to be concentrated.

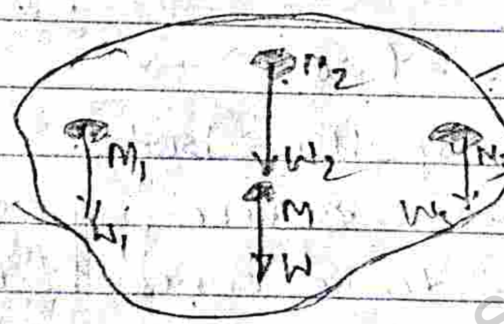
* **Centroid** — The point at which the entire area of a plane lamina or plane figure is supposed to be acting, irrespective of the poisition of the lamina is called centroid.

- It related to Geometry of Body.
- It is geometrical centre.

volume → Area → length/Line.

## Type of Body



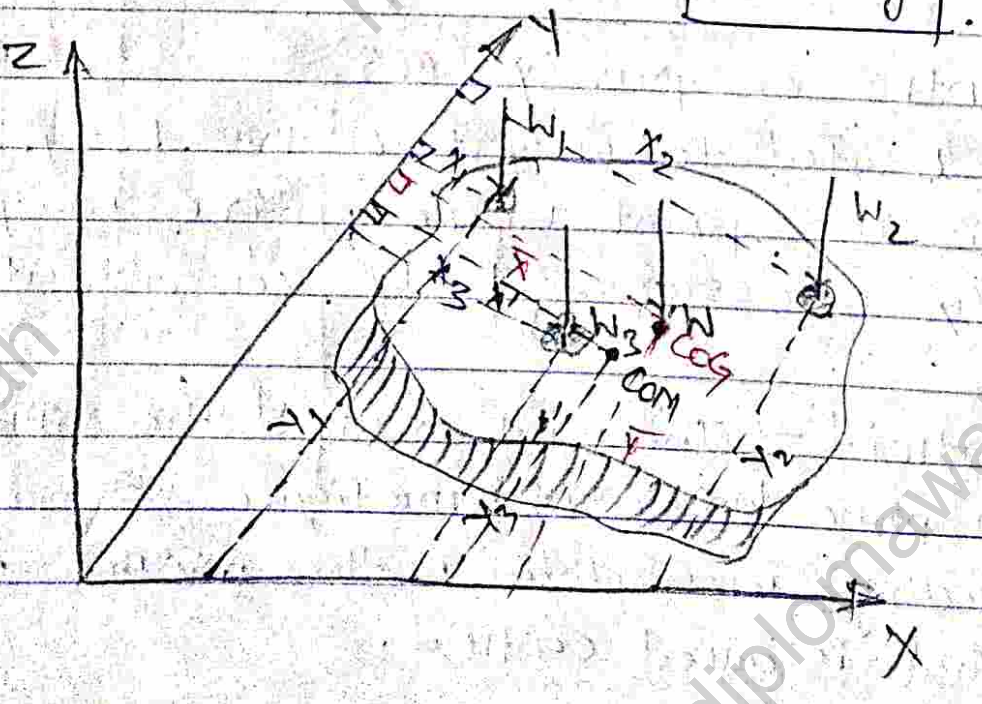




→ Uniform gravitational field (i.e g = constant)

$M = m_1 + m_2 + m_3 + \dots + m_n$

$W = W_1 + W_2 + W_3 + \dots + W_n$

$\boxed{W \propto g}$.



flat horizontal plate –

COG –

By using Varignon's theorem –

- Moment of weight about y-axis.

$$W \cdot \bar{x} = W_1 x_1 + W_2 x_2 + W_3 x_3 + \dots + W_n x_n$$

$$\boxed{\bar{x} = \frac{W_1 x_1 + W_2 x_2 + W_3 x_3 + \dots + W_n x_n}{W_1 + W_2 + W_3 + \dots + W_n}}$$

- Moment of weight about x-axis.

$$W \cdot \bar{y} = W_1 y_1 + W_2 y_2 + W_3 y_3 + \dots W_n y_n$$

$$\boxed{\bar{y} = \frac{W_1 y_1 + W_2 y_2 + W_3 y_3 + \dots + W_n y_n}{W_1 + W_2 + W_3 + \dots + W_n}}$$

COM –

- Moment of mass about y-axis.

$$m x' = m_1 x_1 + m_2 x_2 + m_3 x_3 + \dots + m_n x_n$$

$$\boxed{x' = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \dots + m_n x_n}{m_1 + m_2 + m_3 + \dots + m_n}}$$

- Moment of mass about x-axis.

$$\boxed{y' = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + \dots + m_n y_n}{m_1 + m_2 + m_3 + \dots + m_n}}$$

if body (flate horizontal plate) is in Uniform gravitational field that means value g is constant throughout the body, then

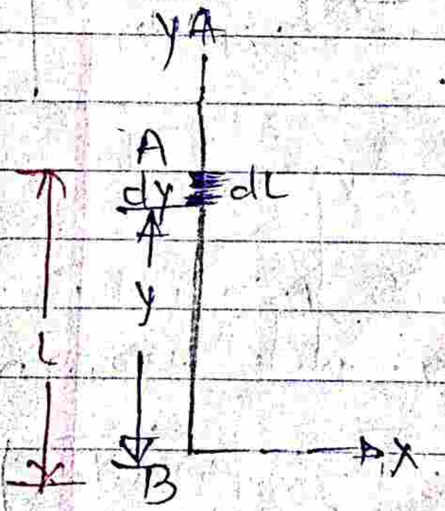
$$\boxed{COG = COM}$$

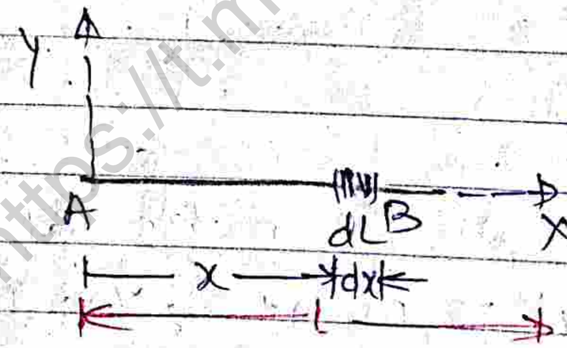
$$\boxed{\bar{x} = x'} \qquad \boxed{\bar{y} = y'}$$

Elemental strip

$$\boxed{\bar{x} = \frac{\int \bar{x}_{ele}\, dL}{\int dL} \qquad \bar{y} = \frac{\int \bar{y}_{ele}\, dL}{\int dL}}$$

for single line / curved shape line.




$dL = dy$

$\bar{x}_{ele} = 0$

$\bar{y}_{ele} = y$

$dL = dx$

$\bar{x}_{ele} = x$

$\bar{y}_{ele} = 0$

COL —

$$\bar{x} = \frac{l_1 x_1 + l_2 x_2 + \cdots + l_n x_n}{l_1 + l_2 + l_3 + \cdots + l_n}$$

$$\bar{y} = \frac{l_1 y_1 + l_2 y_2 + \cdots + l_n y_n}{l_1 + l_2 + l_3 + \cdots + l_n}$$

COA —

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2 + \cdots + A_n x_n}{A_1 + A_2 + A_3 + \cdots + A_n}$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + \cdots + A_n y_n}{A_1 + A_2 + \cdots + A_n}$$

COV —

$$\bar{x} = \frac{V_1 x_1 + V_2 x_2 + \cdots + V_n x_n}{V_1 + V_2 + \cdots + V_n}$$

$$\bar{y} = \frac{V_1 y_1 + V_2 y_2 + \cdots + V_n y_n}{V_1 + V_2 + \cdots + V_n}$$

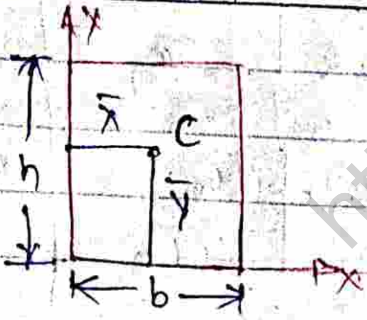
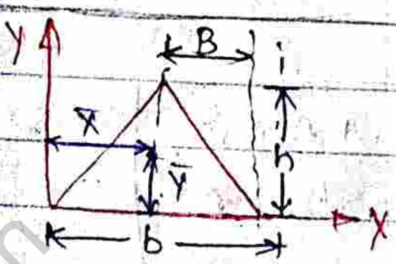
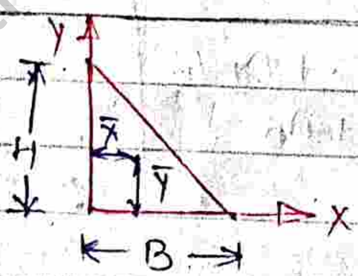
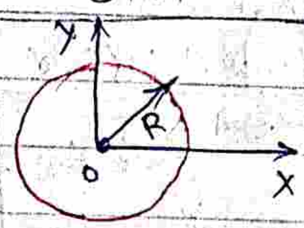
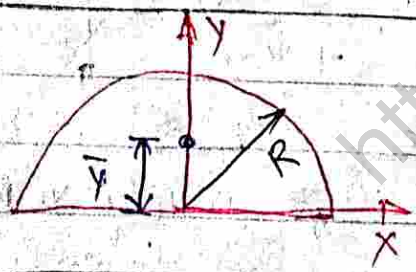
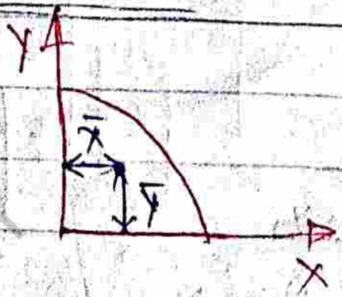
COM —

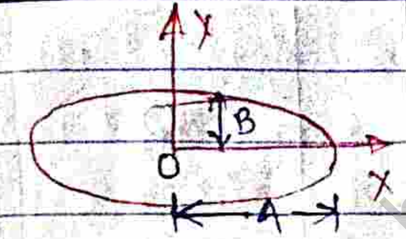
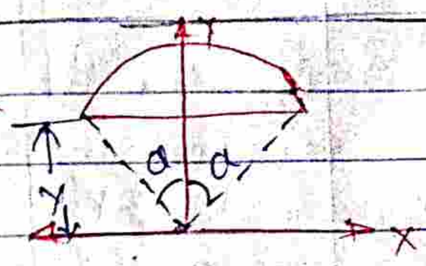
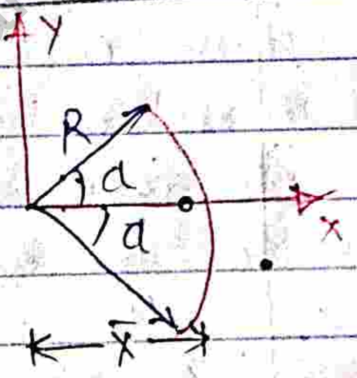
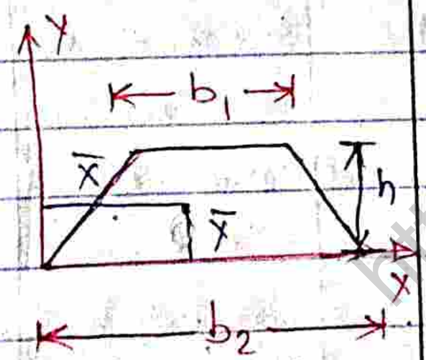
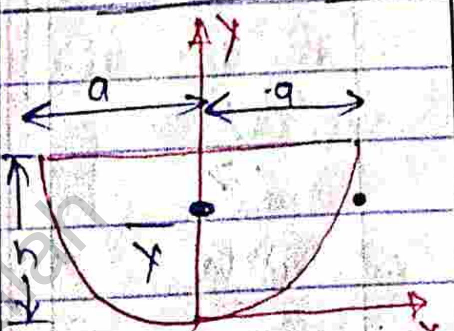
$$\bar{x} = \frac{m_1 x_1 + m_2 x_2 + \cdots + m_n x_n}{m_1 + m_2 + \cdots + m_n} \qquad \bar{y} = \frac{m_1 y_1 + m_2 y_2 + \cdots + m_n y_n}{m_1 + m_2 + \cdots + m_n}$$

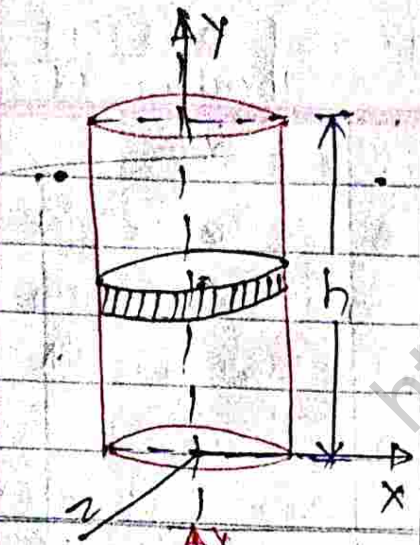
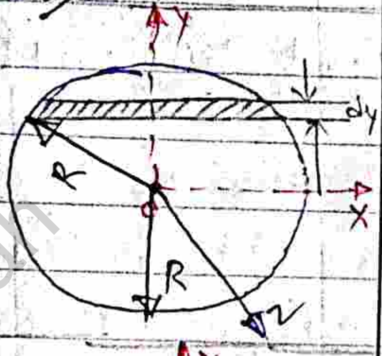
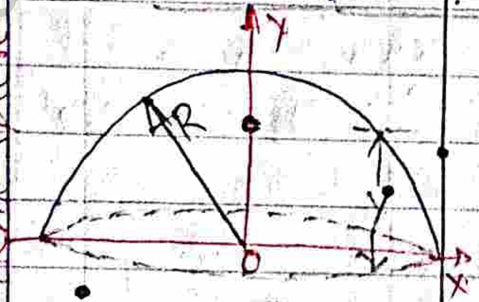
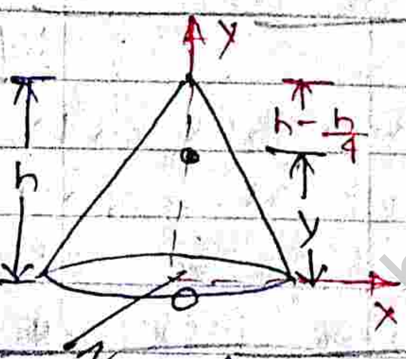
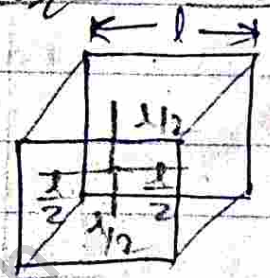
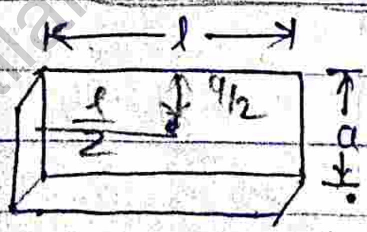
COG —

$$\bar{x} = \frac{W_1 x_1 + W_2 x_2 + \cdots + W_n x_n}{W_1 + W_2 + \cdots + W_n}$$

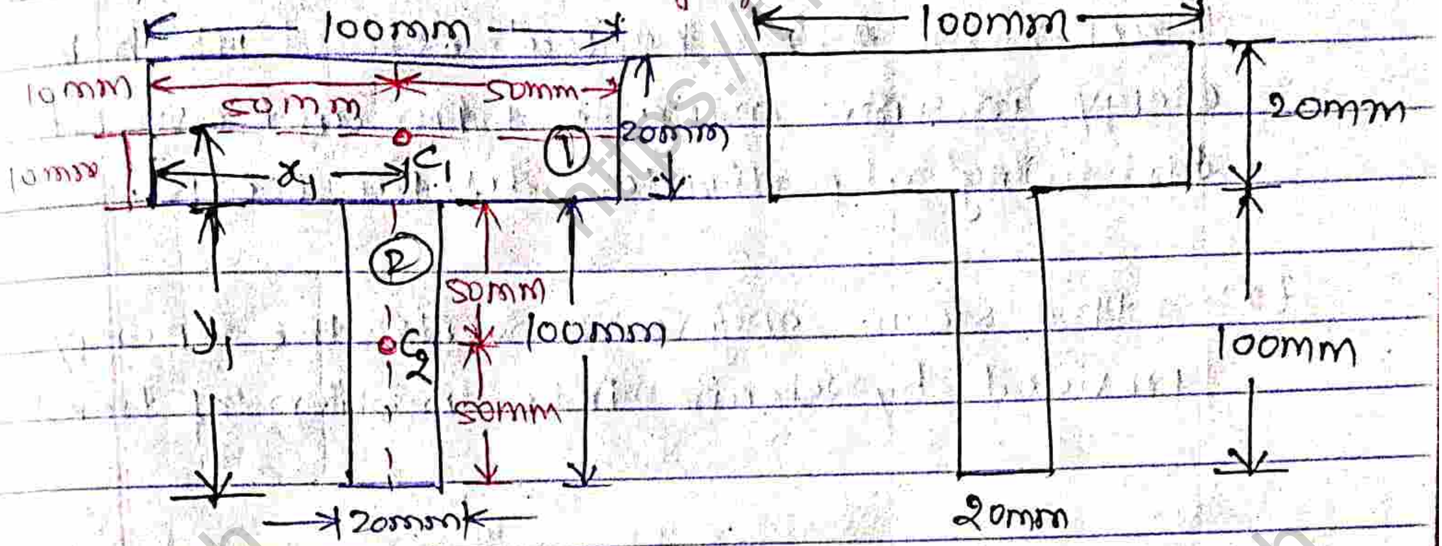
$$\bar{y} = \frac{W_1 y_1 + W_2 y_2 + \cdots + W_n y_n}{W_1 + W_2 + \cdots + W_n}$$

| S.No. | Basic shape | Area | location of centroid $\bar{x}$ | $\bar{y}$ |
|---|---|---|---|---|
| 1. Rectangle |  | $bh$ | $\frac{b}{2}$ | $\frac{h}{2}$ |
| 2. Triangle |  | $\frac{1}{2} \times b \times h$ | $\frac{2b - B}{3}$ | $\frac{h}{3}$ |
| 3. Right-angled Triangle |  | $\frac{1}{2} \times B \times H$ | $\frac{B}{3}$ | $\frac{H}{3}$ |
| 4. Circle |  | $\pi R^2$ | 0 | 0 |
| 5. Semi-circle |  | $\frac{\pi R^2}{2}$ | 0 | $\frac{4R}{3\pi}$ |
| 6. Quarter-circle |  | $\frac{\pi R^2}{4}$ | $\frac{4R}{3\pi}$ | $\frac{4R}{3\pi}$ |

| | Basic Shape | Area | Location of centroid $\bar{x}$ | $\bar{y}$ |
|---|---|---|---|---|
| 7. Ellipse |  | $\pi AB$ | 0 | 0 |
| 8. Circular Segment |  | $R^2a - R^2\sin a \cos a$ | 0 | $\frac{2R}{3}\left(\frac{\sin^3 a}{a-\sin a\cos a}\right)$ |
| 9. Circular Sector |  | $aR^2$ | $\frac{2r\sin a}{3a}$ | 0 |
| 10. Trapezoid |  | $\frac{1}{2}(b_1+b_2)h$ | | $\frac{h}{3}\frac{(2b_1+b_2)}{(b_1+b_2)}$ |
| 11. Parabolic Area |  | $\frac{4ah}{3}$ | 0 | $\frac{3h}{5}$ |

| | | | | |
|---|---|---|---|---|
| 12 cylinder |   | $\pi r^2 h$ | 0 | $\frac{h}{2}$ |
| 13 Sphere |   | $\frac{4}{3}\pi R^3$ | 0 | 0 |
| 14 Hemi-sphere |   | $\frac{2}{3}\pi R^3$ | 0 | $\frac{3}{8}R$ |
| 15 cone |   | $\frac{1}{3}\pi r^2 h$ | 0 | $\frac{h}{4}$ |
| 16 cube |   | $l^3$ | $\frac{l}{2}$ | $\frac{l}{2}$ |
| 17 |   | | $\frac{l}{2}$ | $\frac{a}{2}$ |

Q. Find the centroid of given T-section area.




$A_1 = l \times b = 100 \times 20 = 2000\,mm^2$

$x_1 = 50mm$

$y_1 = 110mm$

$A_2 = l \times b = 100 \times 20 = 2000\,mm^2$

$x_2 = 50mm$

$y_2 = 50mm$

$\bar{X} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2} = \frac{2000 \times 50 + 2000 \times 50}{2000 + 2000} = \frac{100}{2} = 50.$

$\bar{Y} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2} = \frac{2000 \times 110 + 2000 \times 50}{2000 + 2000} = \frac{160}{2} = 80.$

$\boxed{\bar{X} = 50mm, \quad \bar{Y} = 80mm}$

*J.M.S*  Engg. Mechanics

# UNIT-5 SIMPLE LIFTING MACHINE

Machine – A mechanical device which receives energy in some available form and uses it for doing a particular useful work.

Ex:- the steam engine converts the energy provided by steam into motion of translation.

Simple Machine – A machine with the help of which we can lift heavy or very heavy loads by applying comparatively less effort is called Simple Machine.

Ex:- Screw Jack, Nail cutter, Hand pump, Winch crab, Wheels and axles ---etc

characteristics of a simple machine –

1. Has a few or no moving parts.
2. Uses energy to perform work.
3. Works with one movement.
4. makes work easier by using less mechanical effort for moving an object.
5. Uses the concept of spreading force over distan

Compound Machine :- It is a combination of number of simple machine.

Ex:- Cranes, Bull-dozers, Trucks ....

load (W) :- It is the resistance which needs to be overcome.

- Its unit of W is Newton (N).

Effort (P) :- It is quantity which we apply.

or,

The force which is to be applied to a machine to overcome resistance of force or to lift a load.

$W > P$

Mechanical advantage (M.A) — It is defined as the ratio of load or effect.

$$\boxed{M.A = \frac{W}{P}}$$

- M.A doesn't have a unit. It is just a number.
- $M.A > 1$ (Always).

Velocity Ratio (V.R) — It is defined as the ratio of distance travelled by the effort to the corresponding distance travelled by the load.

$$\boxed{V.R = \frac{\text{Distance travelled by the effort}}{\text{Distance travelled by the load}}}$$

- V.R doesn't have a unit. It is just a number.
- V.R > 1 (Always).

Input of a machine — It is defined as the product of effort and distance travelled by effort.

$$\boxed{\text{Input} = P \times x}$$

Where,
$x$ = distance travelled by the effort

Output of a machine — It is defined as the product of load and distance travelled by load.

$$\boxed{\text{Output} = W \times y}$$

Where,
$y$ = distance travelled by the load.

Efficiency of a Machine ($\eta$) — It is defined as the ratio of output of machine to the Input of a machine.

$$\boxed{\eta = \frac{\text{output}}{\text{Input}}}$$

$$\eta = \frac{\text{output}}{\text{Input}} = \frac{W \times y}{P \times x} = \left(\frac{W}{P}\right) \times \frac{1}{\left(\frac{x}{y}\right)}$$

$$\boxed{\eta = \frac{M \cdot A}{V \cdot R}} \qquad \left[\because M \cdot A = \frac{W}{P},\ V \cdot R = \frac{x}{y}\right]$$

Ideal Machine — It is defined as a machine in which there is no friction.

- Efficiency is 100%. $[\because M \cdot A = V \cdot R]$
- Such machine are in impractical.

Actual Machine — It is defined as a machine in which is present.

- Efficiency is less than 100%. $[M \cdot A < V \cdot R]$
- Actual machine are practical machine.

Ideal Effort ($P_i$) — It is that effort which is applied in an ideal machine (No friction).

$$\boxed{M \cdot A = V \cdot R}$$

$$\therefore \frac{W}{P} = V \cdot R$$

$$\boxed{P_i = \frac{W}{V \cdot R}} \longrightarrow \text{Ideal Effort } (P_i)$$

Ideal load ($W_i$) — It is a load which is lifted in an Ideal Machine.

- $MA = VR$ $\qquad [\because \eta = 100\%]$

$$\therefore \frac{W}{P} = V \cdot R$$

$$\boxed{W_i = P \times V \cdot R} \longrightarrow \text{Ideal load } (W_i)$$

Law of machine — It is an equation which gives the relation between effort applied and load lifted.

- It is given by : $\boxed{P = mW + C}$

$P = W m + c$

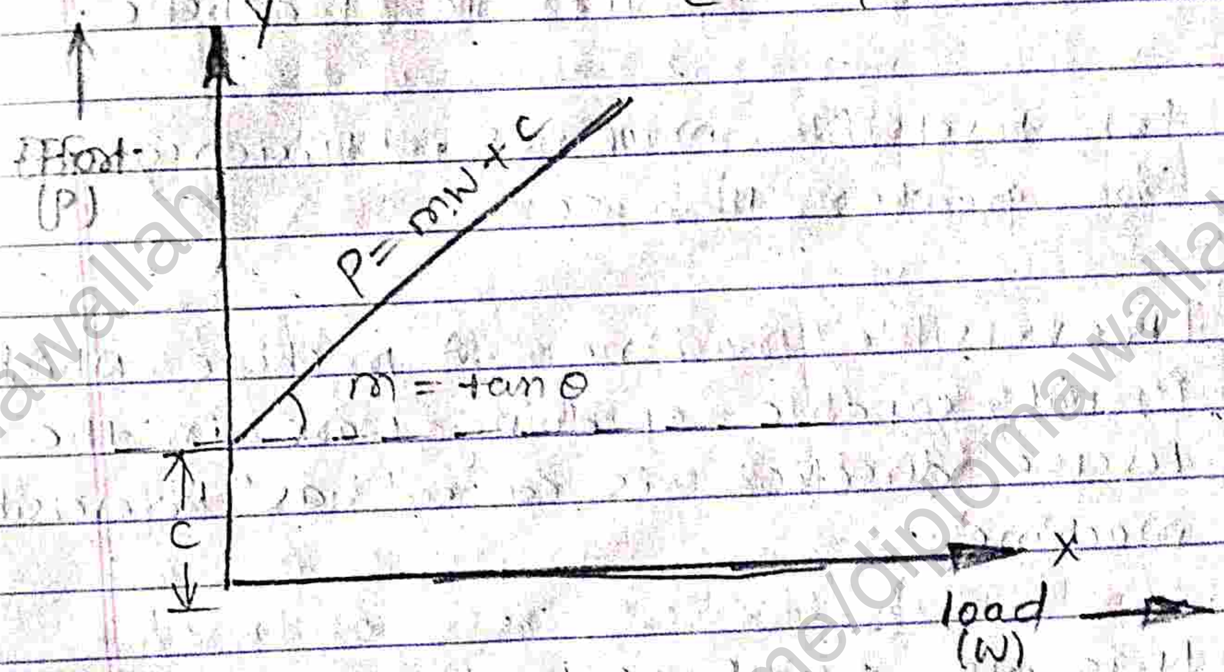
Where,

$P$ = Effort applied

$m$ = slope of line

$W$ = load lifted

$c$ = y - intercept.




Maximum Mechanical Advantage $(M \cdot A)^{max}$ —

It is reciporcal of slope

$$\boxed{M \cdot A^{max} = \frac{1}{m}}$$

Where

$m$ = slope of line

$m = \tan\theta$

Maximum Efficiency — It is defined as the ratio of maximum mechanical advantage $(M \cdot A^{max})$ and Velocity Ratio $(V \cdot R)$.

$$\boxed{\eta_{max} = \frac{M \cdot A^{max}}{V \cdot R}}$$

- **Reversible machine** — A machine which is capable of doing work in the reverse direction i.e even after the removal of effort ; load gets lifted is called "Reversible Machine".

- For reversible machines, Efficiency will be greater than 50%.

**Irreversible machine** — A machine which is not capable of doing work in the reverse direction is called as "Irreversible Machine".

- It is also called as 'self locking Machine'.
- For Irreversible machines, Efficiency will be less than 50%.

**load lost in friction** — It is the difference of Ideal load & actual load.

$$\boxed{W_F = W_i - W}$$

$W_i > W$

**Effort lost in friction** — It is the difference of Actual effort & Ideal effort.

$$\boxed{P_F = P - P_i}$$

$P > P_i$

Q. The velocity ratio of a certain lifting machine is 25. Determine the effort required to lift a load of 900N if efficiency of the machine is 63.8%.

Sol:- $VR = 25$, $W = 900N$, $\eta = 63.8\%$.

$$\% \eta = \frac{M.A}{V.R} \times 100$$

$$63.8 = \frac{M.A}{25}$$

$$M.A = 63.8 \times 25$$

$$M.A = 15.95$$

$$\frac{W}{P} = 15.95$$

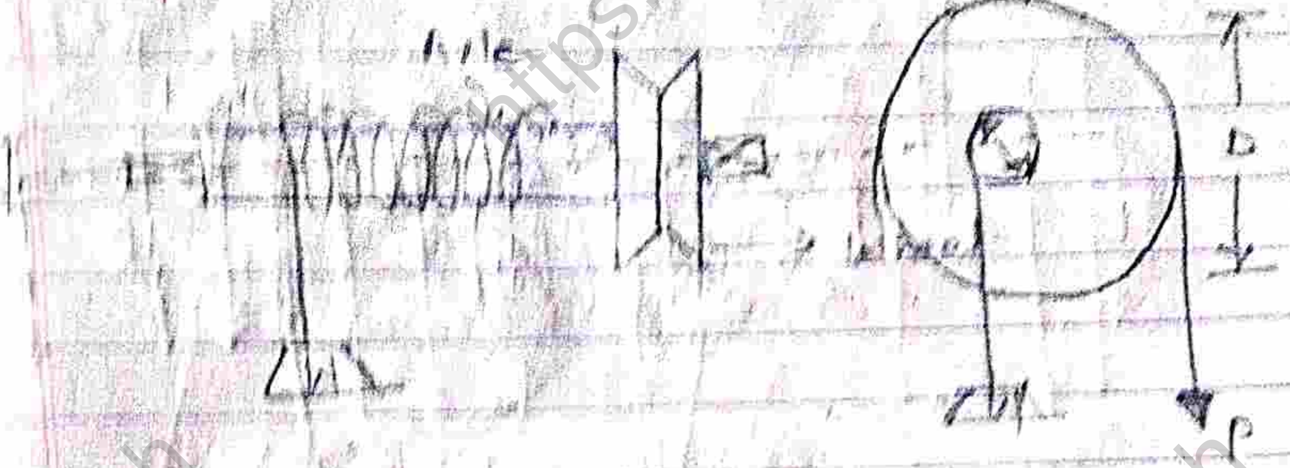
$$\cancel{P = \frac{15.95}{W} = 15.95}$$

$$P = \frac{W}{15.95} = \frac{900}{15.95} = 56.43N.$$

# Stydy of Simple Machine

1. Simple axle and wheel.
2. Differential axle and wheel.
3. weston's differential pulley block.
4. single purchase crab winch.
5. Double purchase crab winch.
6. Worm and worm wheel.
7. Geared pully block
8. Screw Jack
9. Differentail screw Jack.
10. pulleys
    (a) First system of pulleys'
    (b) Second system of pulleys.
    (c) Third system of pulleys.
11. Gear train
12. Host Mechanism.

## Simple axle and wheel




D = Diameter of the effort wheel.
d = Diameter of the load axle.
W = load lifted
P = Effort applied to lift the load.

$$\boxed{V.R = \frac{\text{Distance moved by the effort}}{\text{Distance moved by the load}}}$$

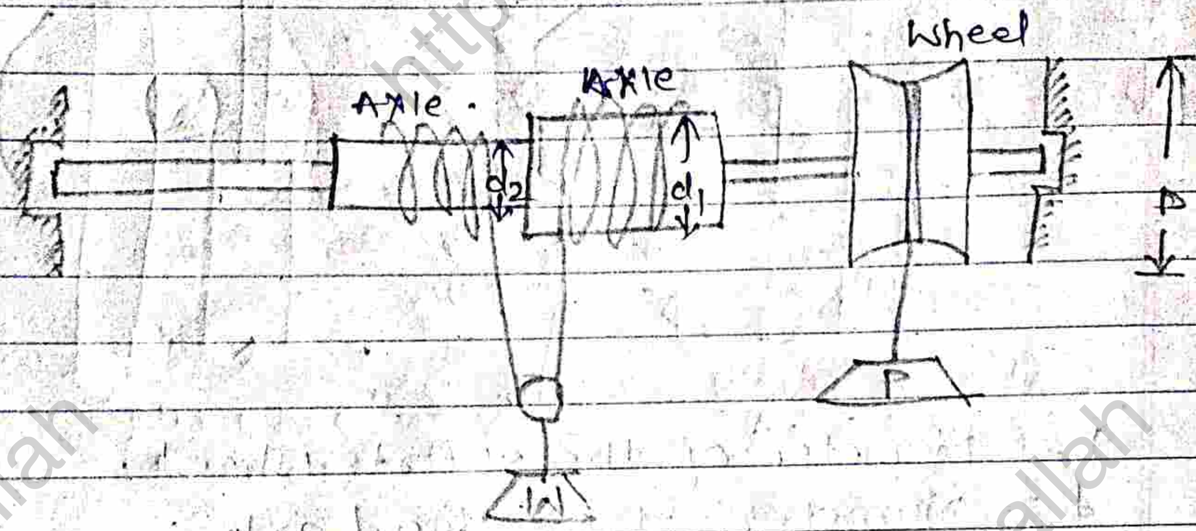
$$V.R = \frac{\pi D}{\pi d} = \frac{D}{d}$$

$$\boxed{V.R = \frac{D}{d}}$$

- If the string wound on the wheel and axle have considerable thickness $t_1$ and $t_2$ respectively.

$$\boxed{V.R = \frac{D+t_1}{d+t_2}}$$

## 2. Differential axle and wheel :




D = Diameter of the ~~effort~~ wheel

$d_1$ & $d_2$ = diameter of the load axle.

$$V.R = \frac{\text{Distance moved by the effort}}{\text{Distance moved by the load}}$$

Now,

Net length wound on axle = $\pi d_1 - \pi d_2$

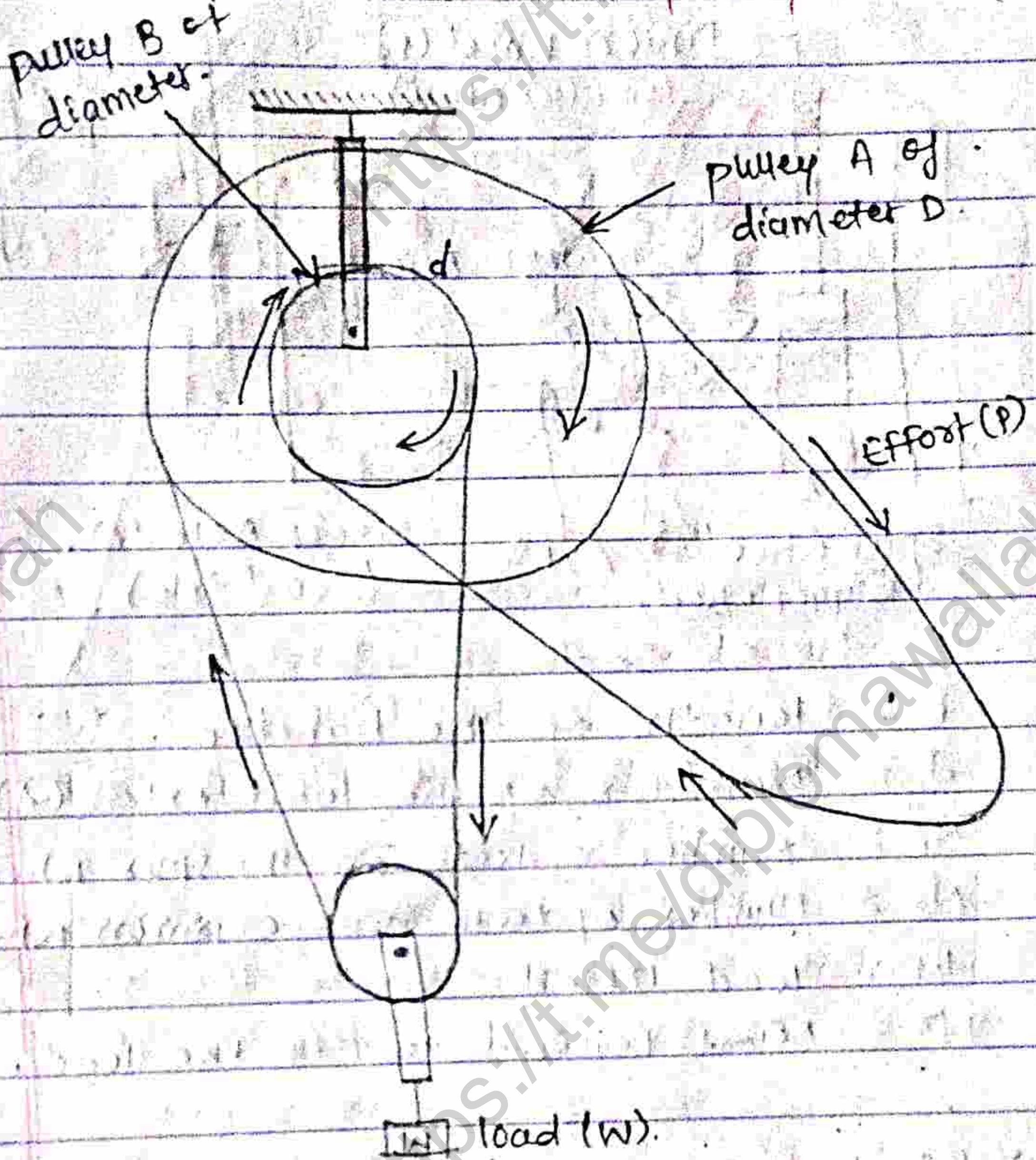
As a movable pulley supports a load W, the load is lifted through half the winding.

Distance moved by load = $\dfrac{\pi d_1 - \pi d_2}{2}$

$$V.R = \frac{\pi D}{\frac{\pi}{2}(d_1 - d_2)} = \frac{2D}{d_1 - d_2}$$

$$\boxed{V.R = \frac{2D}{d_1 - d_2}}$$

3. Weston's differential pulley's block.



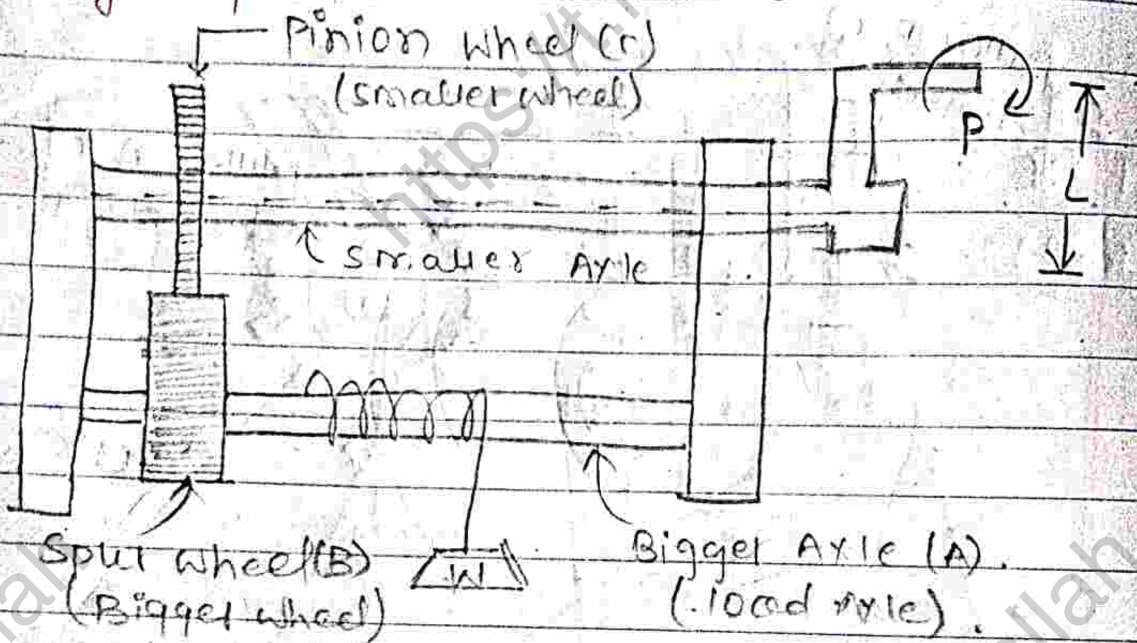

Displacement of the load $= \frac{\pi D - \pi d}{2}$

$$V.R = \frac{\text{Distance moved by the effort}}{\text{Distance moved by the load}}$$

$$V.R = \frac{\pi D}{\frac{\pi(D-d)}{2}}$$

$$\boxed{V.R = \frac{2D}{D-d}}$$

4. Single purchase winch crab



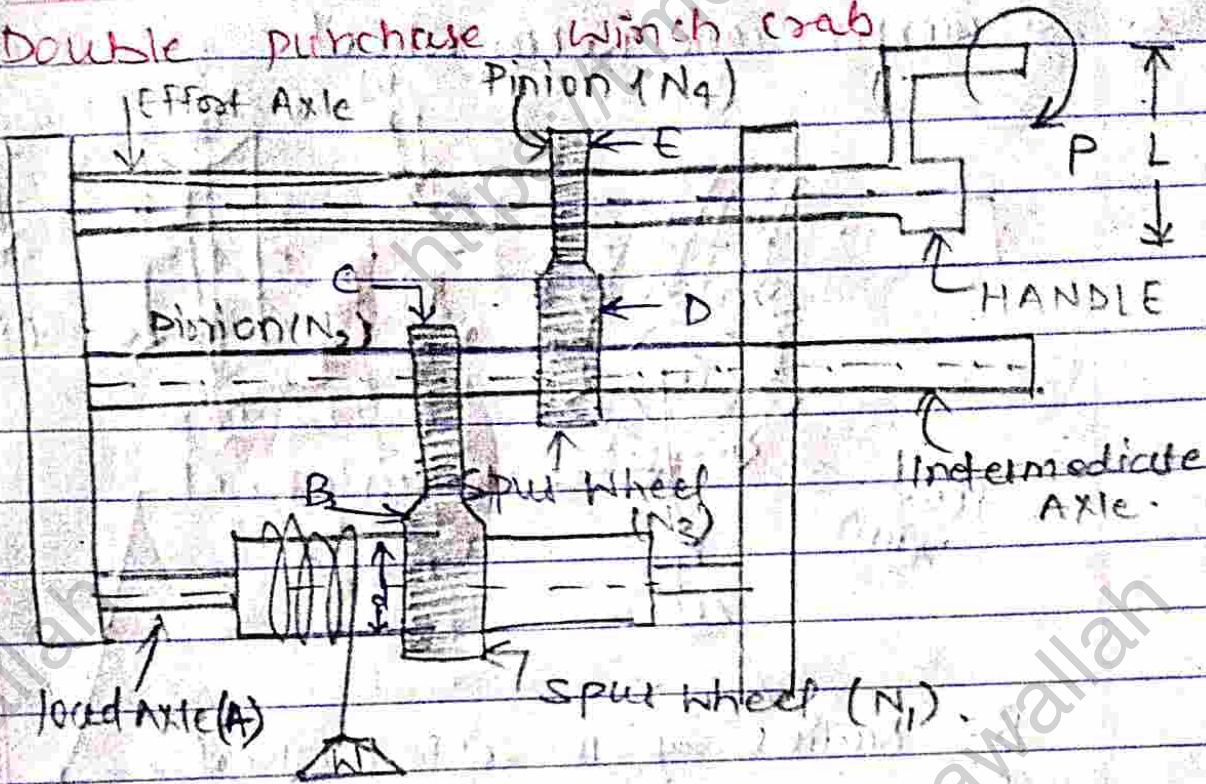

L = length of the handle.
d = Diameter of the load Axle (A)
$N_1$ = Number of teeth on the spur wheel (B)
$N_2$ = Number of teeth on the pinion wheel (C)
W = load lifted
P = Effort required to lift the load.

$$V.R = \frac{\text{Distance moved by Effort}}{\text{Distance moved by load}}$$

$$V.R = \frac{\pi D}{\frac{N_2}{N_1} \times \pi d} \qquad [\because D = 2L]$$

$$\boxed{V.R = \frac{D}{d} \times \frac{N_1}{N_2}}$$

5. Double purchase winch crab



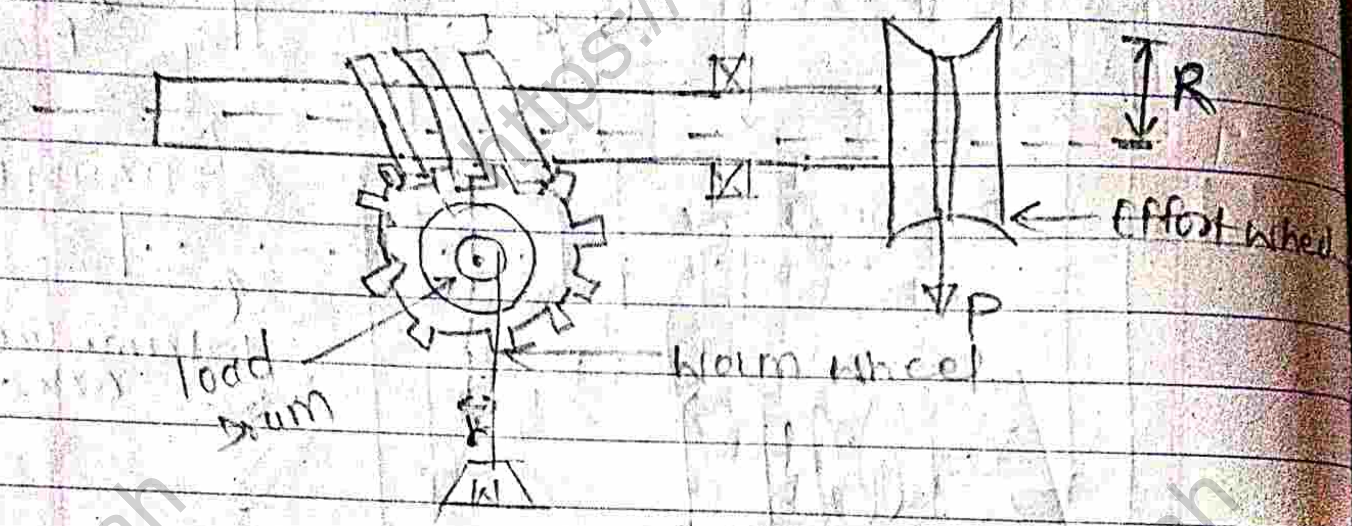

$N_1$ = No. of teeth on the spur wheel on load Axle.
$N_2$ = No. of teeth on the pinion wheel on Intermediate
$N_3$ = No. of teeth on the spur wheel on Intermediate
$N_4$ = No. of teeth on the pinion wheel on effort
$L$ = length of the handle
$d$ = diameter of the load axle
$W$ = load lifted
$P$ = Effort.

$$V.R = \frac{\text{Distance moved by the effort}}{\text{Distance moved by the load}}$$

$$V.R = \frac{2\pi l}{\pi d\left(\frac{N_4}{N_3} \times \frac{N_2}{N_1}\right)}$$

$$\boxed{V.R = \frac{2l}{d} \times \frac{N_1}{N_2} \times \frac{N_3}{N_4}}$$

6. Worm and worm wheel




$R$ = Radius of the effort wheel
$r$ = Radius of the load drum
$T$ = No. of teeth on the worm-wheel.
$W$ = load lifted
$P$ = Effort

Distance through which the load by $\frac{1}{T}$ revolution

$$= \frac{2\pi r}{T}$$

$$V.R = \frac{\text{Distance moved by Effort}}{\text{Distance moved by load}}$$

$$V.R = \frac{2\pi R}{\frac{2\pi r}{T}}$$

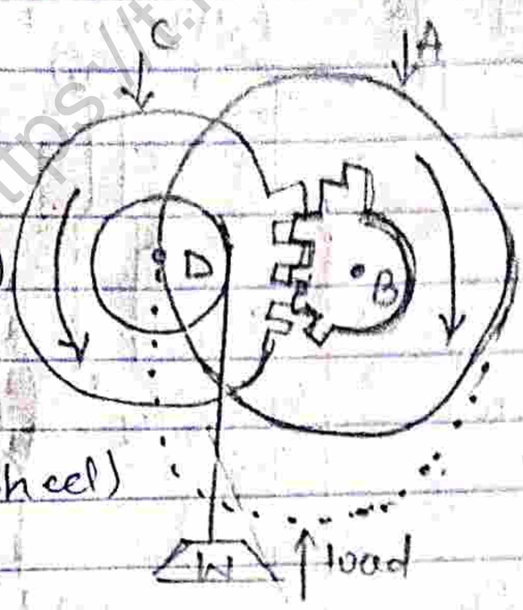
$$\boxed{V.R = \frac{RT}{r}}$$

If the worm is n threaded,

$$\boxed{V.R = \frac{RT}{nr} \text{ or } \frac{LT}{nr}}$$

7. Gread pulley block

A = cog (wheel) (Effort wheel)
B = pinion wheel
C = spur wheel
D = cog wheel (load wheel)




$N_1$ = No. of cogs on the effort wheel A
$N_2$ = No. of teeth on the pinion wheel B
$N_3$ = No. of teeth on the spur wheel C
$N_4$ = No. of cogs on the load wheel D

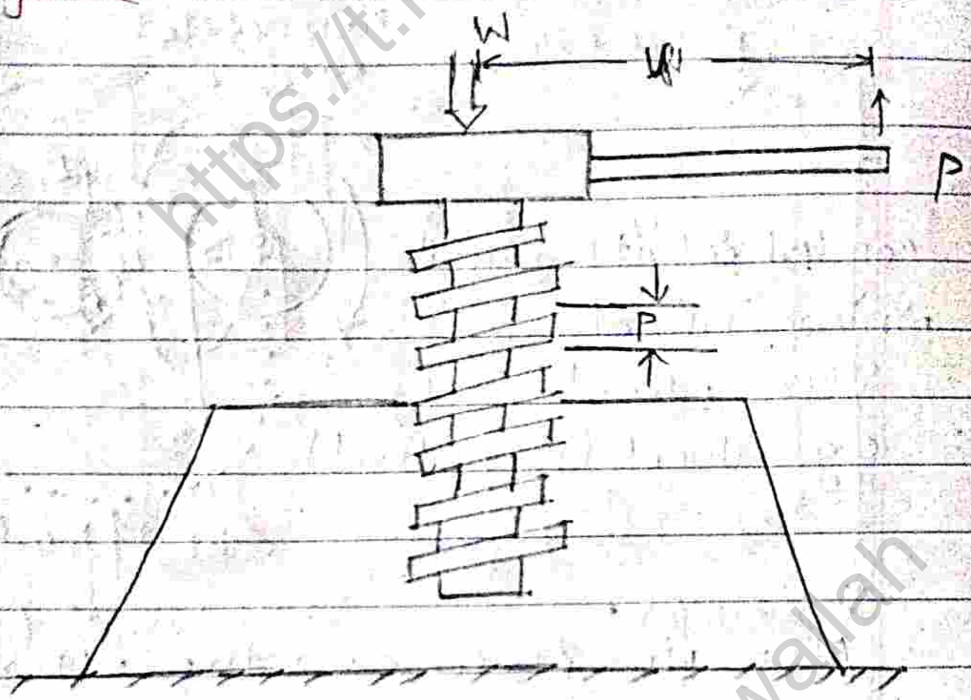
Rotate the spur wheel C through $\frac{N_2}{N_3}$ revolution.

$$V.R = \frac{\text{Distance moved by effort}}{\text{Distance moved by the load}}$$

$$V.R = \frac{N_1}{\frac{N_2}{N_3} \times N_4} = \frac{N_1}{N_2} \times \frac{N_3}{N_4}$$

$$\boxed{V.R = \frac{D}{d} \times \frac{N_3}{N_2}}$$

8. Screw Jack




L = length of the handle.
P = pitch of the screw.

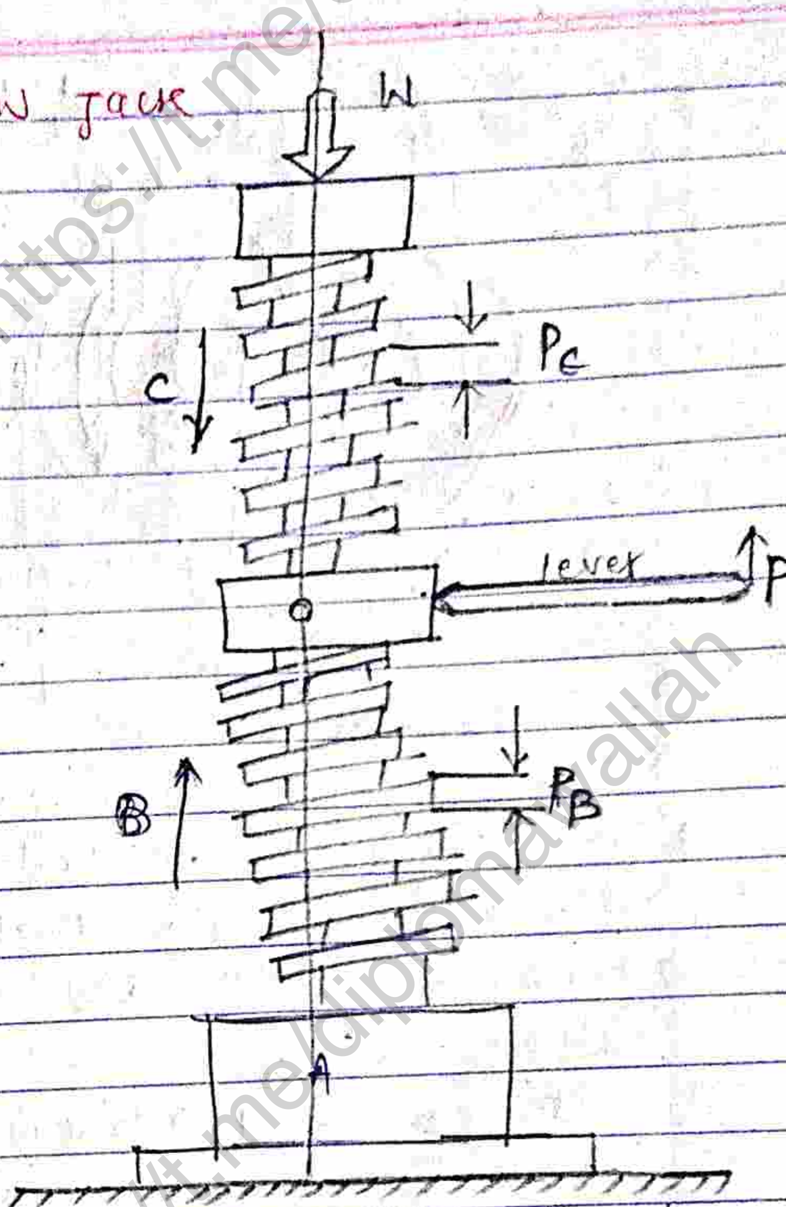
$$V.R = \frac{\text{Distance moved by the effort}}{\text{Distance moved by the load}}$$

$$\boxed{V.R = \frac{2\pi L}{P}}$$

- if Instead of a handle of length L, an effort wheel of radius R is attached

$$\boxed{V.R = \frac{2\pi R}{P}}$$

9. Differential Screw Jack




$P_B$ = pitch of threads of spindle B

$P_C$ = pitch of threads of spindle C.

for one revolution of the lever. Displacement of P = $2\pi l$.

$$V \cdot R = \frac{\text{Distance moved by the effort}}{\text{Distance moved by the load}}$$

$$\boxed{V \cdot R = \frac{2\pi l}{P_B - P_C}}$$